Math 0220

Fall 2017

Quiz 1

S o l u t i o n s

---

1. Simplify expressions

(a) (3 points) $\dfrac{2^7\, x^3\, \sqrt[4]{x}}{32\, x^{5/2}}$

Solution: $\dfrac{2^7\, x^3\, \sqrt[4]{x}}{32\, x^{5/2}} = \dfrac{2^7\, x^{13/4}}{2^5\, x^{10/4}} = 4\, x^{3/4}\,,$ **+2 pts**

when $x \neq 0$. **+1 pt**

(b) (3 points) $\dfrac{x^2 - 5x + 6}{|x-3|}$

Solution: $\dfrac{x^2 - 5x + 6}{|x-3|} = \dfrac{(x-2)(x-3)}{|x-3|},$ **+1 pt**

$$= \begin{cases} \dfrac{(x-2)(x-3)}{x-3}, & x-3 > 0 \\ \dfrac{(x-2)(x-3)}{-(x-3)}, & x-3 < 0 \end{cases} = \begin{cases} x-2, & x > 3 \\ -x+2, & x < 3 \end{cases}$$ **+2 pts**

(c) (3 points) $\dfrac{2\sin 2x}{4\sin x}$

Solution: $\dfrac{2\sin 2x}{4\sin x} = \dfrac{2 \cdot 2\sin x \cos x}{4\sin x} = \cos x,$ **+2 pts**

when $x \neq \pi k,\ k = 0, \pm 1, \pm 2, \ldots$ (i.e. $\sin x \neq 0$). **+1 pt**

2. (3 points) Find the volume V of the cone with radius of the base $r = 3$ cm and hight of $h = 4$ cm.

Solution: $V = \dfrac{1}{3}\pi r^2 h,$ **+1 pt**

$= \frac{1}{3}\pi \cdot 3 \cdot 3 \cdot 4$ **+1 pt**

$= 12\pi \text{ cm}^3.$ **+1 pt**

3. (3 points) Solve the equation $\sin 2x = -\frac{1}{2}$, if $\frac{3}{4}\pi \le x \le \pi$.

Solution: $\sin 2x = -\frac{1}{2}$

$\Leftrightarrow \quad 2x = -\frac{1}{6}\pi + \pi k, \ k = 0, \pm 1, \pm 2, \ldots \quad \text{or} \quad 2x = -\frac{5}{6}\pi + \pi n, \ n = 0, \pm 1, \pm 2, \ldots$ **+1 pt**

$\Leftrightarrow \quad x = -\frac{\pi}{12} + \frac{\pi k}{2}, \ k = 0, \pm 1, \pm 2, \ldots \quad \text{or} \quad x = -\frac{5\pi}{12} + \frac{\pi n}{2}, \ n = 0, \pm 1, \pm 2, \ldots$ **+1 pt**

The first set when $k = 2$ gives $x = \frac{11}{12}\pi$ which lies in the interval $\left[\frac{3}{4}\pi, \pi\right]$.

Answer: $x = \frac{11}{12}\pi$. **+1 pt**

bonus problem [3 points extra] Simplify the expression $\dfrac{(\sqrt{x}+3)(x-6\sqrt{x}+9)}{9-x}$

Solution: $\dfrac{(\sqrt{x}+3)(x-6\sqrt{x}+9)}{9-x} = \dfrac{(\sqrt{x}+3)(\sqrt{x}-3)^2}{(3+\sqrt{x})(3-\sqrt{x})} =$ **+1 pt**

$= -(\sqrt{x}-3) = 3 - \sqrt{x},$ **+1 pt**

$x \in [0, 9) \cup (9, \infty).$ **+1 pt**