Math 0220

Fall 2017

# Quiz 2
## Solutions

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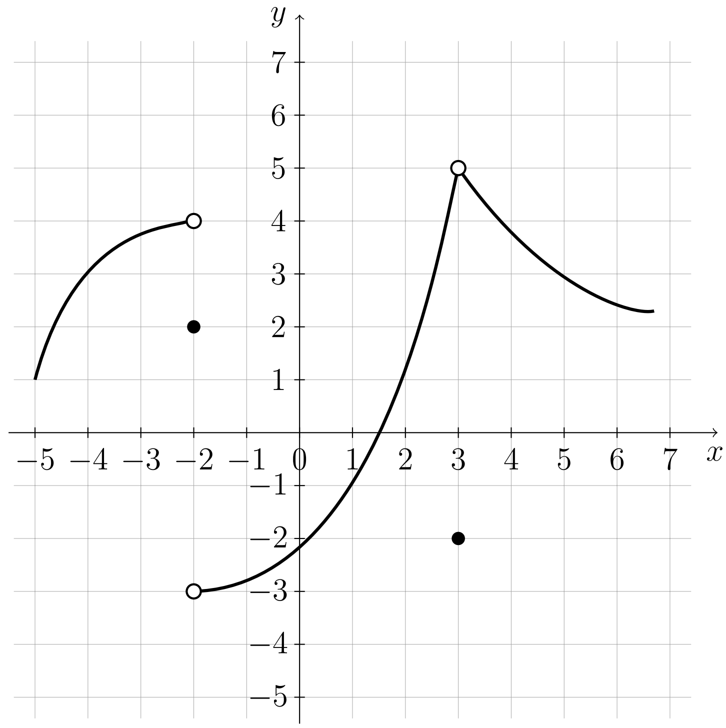
1. For the function $f$ whose graph is given, state the value of each equity, if it exists. If it does not exist, explain why.




(a) $\lim_{x\to -2^-} f(x)$ *Solution:* $\lim_{x\to -2^-} f(x) = 4.$ **+1 pt**

(b) $\lim_{x\to -2^+} f(x)$ *Solution:* $\lim_{x\to -2^+} f(x) = -3.$ **+1 pt**

(c) $\lim_{x\to -2} f(x)$ *Solution:* $\lim_{x\to -2} f(x)$ DNE because $\lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x)$. **+1 pt**

(d) $f(-2)$ *Solution:* $f(-2) = 2.$ **+1 pt**

(e) $\lim_{x\to 3} f(x)$ *Solution:* $\lim_{x\to 3} f(x) = 5.$ **+1 pt**

(f) $f(3)$ *Solution:* $f(3) = -2.$ **+1 pt**

2. Evaluate the limit, if it exists $\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x}$

*Solution:* $\lim_{x\to 0} \frac{\sqrt{4+x}-2}{x} = \lim_{x\to 0} \frac{\sqrt{4+x}-2}{x} \cdot \frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$ **+1 pt**

$= \lim_{x\to 0} \frac{4+x-4}{x\left(\sqrt{4+x}+2\right)} = \lim_{x\to 0} \frac{x}{x\left(\sqrt{4+x}+2\right)}$ **+1 pt**

$= \lim_{x\to 0} \frac{1}{\sqrt{4+x}+2} \overset{DSP}{=} \lim_{x\to 0} \frac{1}{\sqrt{4+0}+2} = \frac{1}{4}.$ **+1 pt**

3. Explain why the function

$$f(x) = \begin{cases} \dfrac{x+2}{|x+2|} & \text{if } x \neq -2 \\ -1 & \text{if } x = -2 \end{cases}$$

is discontinuous at $x = -2$. Is the discontinuity removable? State the reason why.

*Solution:* $\lim_{x\to -2^-} f(x) = \lim_{x\to -2^-} \frac{x+2}{|x+2|} = \lim_{x\to -2^-} \frac{x+2}{-(x+2)} = \lim_{x\to -2^-} (-1) = -1.$ **+1 pt**

$\lim_{x\to -2^+} f(x) = \lim_{x\to -2^+} \frac{x+2}{|x+2|} = \lim_{x\to -2^+} \frac{x+2}{x+2} = \lim_{x\to -2^-} 1 = 1$ **+1 pt**

$\lim_{x\to -2^-} f(x) \neq \lim_{x\to -2^+} f(x)$. Hence, the function is discontinuous at $x = -2$. **+1 pt**

$\lim_{x\to -2} f(x)$ DNE. Therefore, the discontinuity is not removable. **+1 pt**

bonus problem Is there a number that exactly 1 more than its cube?

*Solution:* We assume that such a number exists and denote it by $x$.

Then the problem can be described as: Is there a solution to the equation $x^3 + 1 = x$?

In other words, is there a root of the polynomial $f(x) = x^3 - x + 1$?

$f(-2) = -5 < 0$, $f(0) = 1 > 0$ and $f(x)$ is continuous on the closed interval $[-2, 0]$.

By the Intermediate Value Theorem there is a number $c \in (-2, 0)$ such that $f(c) = 0$. So, $c$ is a root that solves the problem.

Therefore, such a number exists.