Math 0220 **Quiz 3**

Fall 2017 **Solutions**

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1. Find the limit

(a) $\lim\limits_{x\to(-\pi/2)^-} \sec x$

Solution: $\sec x$ is getting larger and larger as x approaches $-\frac{\pi}{2}$ **+1 pt**

$\sec x < 0$ when $-\pi < x < -\frac{\pi}{2}$ **+1 pt**

Therefore $\lim\limits_{x\to(-\pi/2)^-} \sec x = -\infty$. **+1 pt**

(b) $\lim\limits_{x\to\infty} \left(\sqrt{x^2-x} - x\right)$

Solution: $\lim\limits_{x\to\infty} \left(\sqrt{x^2-x} - x\right) = \lim\limits_{x\to\infty} \left(\sqrt{x^2-x} - x\right) \cdot \dfrac{\sqrt{x^2-x}+x}{\sqrt{x^2-x}+x}$ **+1 pt**

$= \lim\limits_{x\to\infty} \dfrac{x^2-x-x^2}{\sqrt{x^2-x}+x} = \lim\limits_{x\to\infty} \dfrac{-x}{\sqrt{x^2-x}+x}$ **+1 pt**

$= \lim\limits_{x\to\infty} \dfrac{-x}{\sqrt{x^2-x}+x} \cdot \dfrac{\frac{1}{x}}{\frac{1}{x}}$ **+1 pt**

$= \lim\limits_{x\to\infty} \dfrac{-1}{\sqrt{1-\frac{1}{x}}+1}$ **+1 pt**

$= \dfrac{-1}{\sqrt{1-\lim\limits_{x\to\infty}\frac{1}{x}}+1} = \dfrac{-1}{\sqrt{1-0}+1} = \dfrac{-1}{1+1} = -\frac{1}{2}.$ **+1 pt**

2. Using limit find the slope of the tangent line to the curve $y = 1+x^2$ at the point $(-2, 5)$.

Solution: $x = -2$. Let $f(x) = 1 + x^2$. **+1 pt**

The slope is $m = \lim\limits_{h\to 0} \dfrac{f(-2+h) - f(-2)}{h}$ **+1 pt**

$= \lim\limits_{h\to 0} \dfrac{1+(-2+h)^2 - (1+(-2)^2)}{h} = \lim\limits_{h\to 0} \dfrac{1+4-4h+h^2-1-4}{h}.$ **+1 pt**

$= \lim\limits_{h\to 0} \dfrac{-4h+h^2}{h} = \lim\limits_{h\to 0}(-4+h) = -4.$ **+1 pt**

3. The graph of the function f is drawn. Sketch the graph of f' below it.

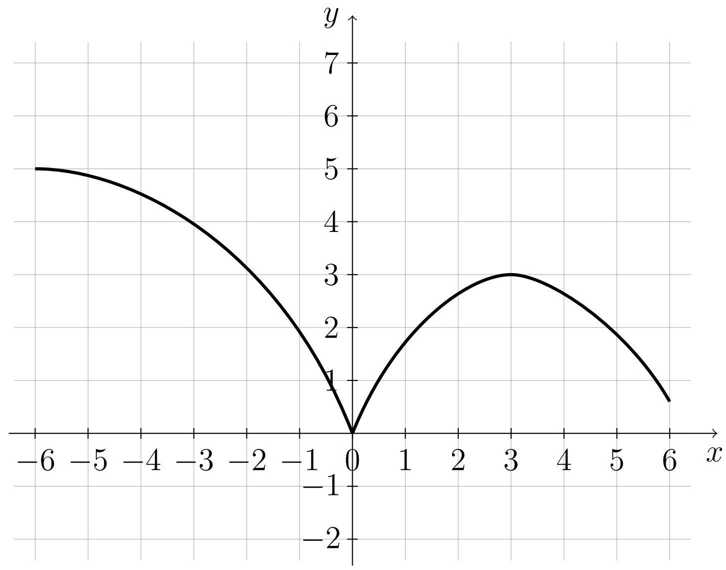

Figure 1: Graph of f.

Solution:

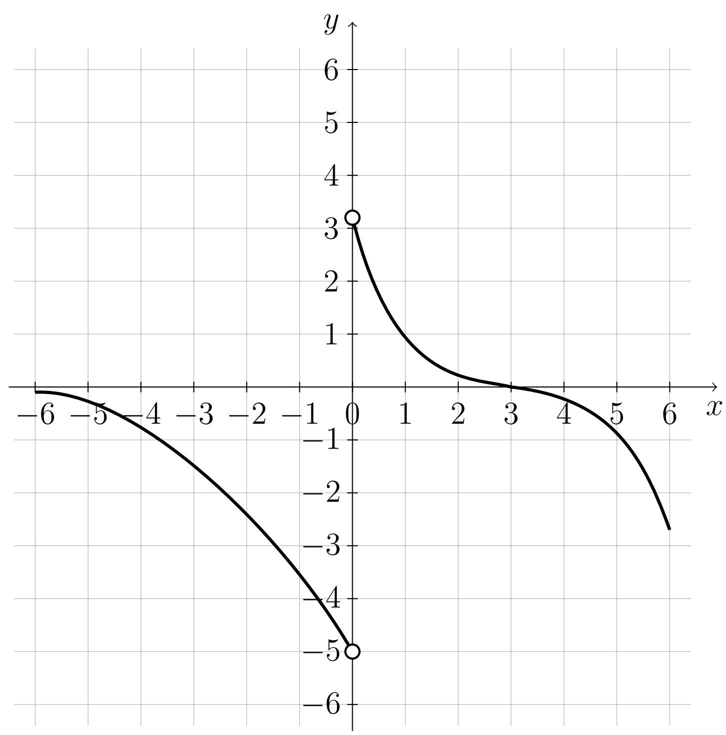

Figure 2: Graph of f'.

bonus problem Find a formula of a function f that satisfies the following conditions

$$\lim_{x\to-\infty} f(x) = 0, \qquad \lim_{x\to+\infty} f(x) = 0, \qquad \lim_{x\to 0} f(x) = -\infty,$$

$$f(2) = 0, \qquad \lim_{x\to 3^-} f(x) = \infty, \qquad \lim_{x\to 3^+} f(x) = -\infty.$$

Solution: $f(x) = \dfrac{2-x}{x^2(x-3)}$