Math 0220

Fall 2017

Quiz 4

S o l u t i o n s

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1. Find y'. Do not simplify your answer.

(a) $y(x) = e^{x\cos x}$

Solution: $y'(x) = (\cos x - x\sin x)\, e^{x\cos x}$ **+1 pt**

(b) $y(x) = \dfrac{2}{\ln x}$

Solution: $y(x) = 2\,(\ln x)^{-1}$ **+1 pt**

$y'(x) = 2(-1)\,(\ln x)^{-2} \cdot \dfrac{1}{x}$ **+1 pt**

$y'(x) = -\dfrac{2}{x\,(\ln x)^2}$ **+1 pt**

(c) $y(x) = x^{3x}$

Solution: $\ln y = 3x \ln x$ **+1 pt**

$\dfrac{y'}{y} = 3\ln x + \dfrac{3x}{x} = 3\ln x + 3$ **+1 pt**

$y'(x) = x^{3x}\,(3\ln x + 3).$ **+1 pt**

(d) $y(x) = \sin^{-1}(3x-2)$

Solution: $y' = \dfrac{3}{\sqrt{1-(3x-2)^2}}$ **+1 pt**

(e) $y(x) = \tanh^{-1}(3x-2)$

Solution: $y' = \dfrac{3}{1-(3x-2)^2}$ **+1 pt**

2. Using l'Hospital's Rule find the limit.

(a) $\displaystyle\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}}$

Solution: $\displaystyle\lim_{x\to\infty} \frac{\ln x}{\sqrt{x}} \underset{\text{"}\frac{\infty}{\infty}\text{"}}{\overset{H}{=}} \lim_{x\to\infty} \frac{1/x}{1/(2\sqrt{x})} = 0$ **+1 pt**

$\displaystyle= \lim_{x\to\infty} \frac{1/x}{1/(2\sqrt{x})} \cdot \frac{2x}{2x} = \lim_{x\to\infty} \frac{2}{\sqrt{x}}$ **+1 pt**

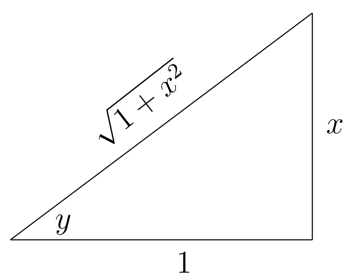
(b) $\displaystyle\lim_{x\to 0} \frac{4^x - 3^x}{x}$

Solution: $\displaystyle\lim_{x\to 0} \frac{4^x - 3^x}{x} \underset{\text{"}\frac{0}{0}\text{"}}{\overset{H}{=}} \lim_{x\to 0} \frac{4^x \ln 4 - 3^x \ln 3}{1} = \ln 4 - \ln 3$ **+1 pt**

$\displaystyle= \lim_{x\to\infty} \frac{1/x}{1/(2\sqrt{x})} \cdot \frac{2x}{2x} = \lim_{x\to\infty} \frac{2}{\sqrt{x}}$ **+1 pt**

bonus problem Simplify the expression $\cos(2\tan^{-1} x)$.

Solution: Denote $y = \tan^{-1} x$. Then $\cos(2\tan^{-1} x) = \cos 2y = (\cos y)^2 - (\sin y)^2$.



$x = \tan y$, $\quad \cos y = \dfrac{1}{\sqrt{1+x^2}}$, $\quad \sin y = \dfrac{x}{\sqrt{1+x^2}}$

Therefore

$$\cos(2\tan^{-1} x) = \frac{1}{1+x^2} - \frac{x^2}{1+x^2} = \frac{1-x^2}{1+x^2}$$