Math 0220 **Quiz 5**

Fall 2017 **S o l u t i o n s**

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1. Find the absolute maximum and absolute minimum values of the function

$f(t) = 2\sqrt{x} - x$ when $0 \le x \le 9$. Justify your answer.

*Solution:* $f$ is continuous and the interval $[0, 9]$ is closed. So, by the Extreme Value Theorem the function attains its absolute maximum and absolute minimum values.

CNs: $f'(x) = \dfrac{1}{\sqrt{x}} - 1 = \dfrac{1-\sqrt{x}}{\sqrt{x}}$. **+1 pt**

$f'(x) = 0 \quad \Leftrightarrow \quad \sqrt{x} - 1 = 0 \quad \Leftrightarrow \quad x = 1;$ **+1 pt**

$f'(x)$ DNE when $\sqrt{x} = 0 \quad \Leftrightarrow \quad x = 0.$ **+1 pt**

CNs are 0 and 1. End points are 0 and 9. **+1 pt**

$f(0) = 0,\ f(1) = 1,\ f(9) = 6 - 9 = -3.$

The absolute maximum value is 1 and the absolute minimum value is $-3$. **+1 pt**

2. For the function $f(x) = x^3 - 3x + 1$

(a) Find intervals on which $f$ is increasing or decreasing.

*Solution:* $f'(x) = 3x^2 - 3$ **+1 pt**

$f'(x) = 0 \quad \Leftrightarrow \quad 3x^2 - 3x = 0 \quad \Leftrightarrow \quad 3(x+1)(x-1) = 0$

$\Leftrightarrow \quad x = -1$ or $x = 1$ **+1 pt**

$f'(x) > 0$ when $x < -1$ or $x > 1$

$f'(x) < 0$ when $-1 < x < 1$ **+1 pt**

Therefore, $f(x)$ is increasing on the intervals $(-\infty, -1)$ and $(1, \infty)$

$f(x)$ is decreasing on the interval $(-1, 1)$. **+1 pt**

(b) Find local maximum and local minimum values of $f$.

*Solution:* CNs are $x = -1$ and $x = 1$ **+1 pt**

$f(x)$ has local maximum value at $x = -1$.

The local maximum value is $f(-1) = -1 + 3 + 1 = 3$ **+1 pt**

$f(x)$ has local minimum value at $x = 1$.

The local maximum value is $f(1) = 1 - 3 + 1 = -1$ **+1 pt**

(c) Find intervals of concavity, types of concavity, and inflection points.

*Solution:* $f''(x) = 6x$ **+1 pt**

$f''(x) = 0 \quad \Leftrightarrow \quad x = 0 \; f(0) = 1.$ IP is $(0, 1)$ **+1 pt**

$f''(x) > 0$ when $x > 0$, $f''(x) < 0$ when $x < 0$

Therefore, $f(x)$ is concave up on the interval $(0, \infty)$

and is concave down on the interval $(-\infty, 0)$. **+1 pt**

bonus problem Suppose that $f(1) = -1$ and $f'(x) \leq 2$ for all $x$. How large can $f(4)$ possibly be?

*Solution:* $f$ is differentiable everywhere. We can apply the Mean Value Theorem on the interval $[1, 4]$. **+1 pt**

There exists a number $c$ from the interval such that

$f(4) - f(1) = f'(c)(4 - 1) \quad \Leftrightarrow \quad f(4) + 1 = 3f'(c) \quad \Leftrightarrow \quad f(4) = -1 + 3f'(c).$ **+2 pts**

$f'(c) \leq 2 \quad \Rightarrow \quad f(4) \leq -1 + 3 \cdot 2 = -1 + 6 = 5.$ **+1 pt**

Therefore the largest possible value for $f(4)$ is 5. **+1 pt**