Math 0220 **Quiz 6**

Fall 2017 **S o l u t i o n s**

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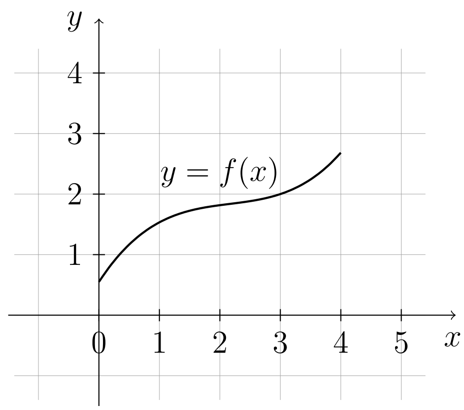
1. The graph of the function is given with $f(0) = 0.7$, $f(1) = 1.5$, $f(2) = 1.8$, $f(3) = 2$, $f(4) = 2.7$.

Estimate $\int_0^4 f(x)\,dx$ using four subintervals with



(a) right endpoints;

Solution: $\Delta x = \frac{4-0}{4} = 1$ **+1 pt**

$\int_0^4 f(x)\,dx \approx R_4 = 1 \cdot (f(1) + f(2) + f(3) + f(4))$ **+1 pt**

$= 1.5 + 1.8 + 2 + 2.7 = 8$ **+1 pt**

(b) left endpoints.

Solution: $\int_0^4 f(x)\,dx \approx L_4 = 1 \cdot (f(0) + f(1) + f(2) + f(3))$ **+1 pt**

$= 0.7 + 1.5 + 1.8 + 2 = 6$ **+1 pt**

2. Evaluate the integral $\int_0^4 \frac{x-1}{\sqrt{x}}\,dx$

Solution: $\displaystyle\int_{-4}^{4} \frac{x-1}{\sqrt{x}}\,dx = \int_0^4 \left(x^{1/2} - x^{-1/2}\right)\,dx$ **+2 pts**

$= \left[\frac{2}{3}x^{3/2} - 2x^{1/2}\right]_0^4$ **+1 pt**

$= \frac{2}{3}\cdot 8 - 2\cdot 2$ **+1 pt**

$= \frac{16}{3} - 4 = \frac{4}{3}$ **+1 pt**

3. Find the average value of the function $f(x) = \dfrac{4x}{x^2-3}$ over the interval $[2,4]$.

Solution: $\displaystyle f_{ave} = \frac{1}{4-2}\int_2^4 \frac{4x}{x^2-3}\,dx = \int_2^4 \frac{2x}{x^2-3}\,dx$ **+1 pt**

u-substitution $u = x^2 - 3,\ du = 2x\,dx,\ 2x\,dx = du,$

$u(2) = 1,\ u(4) = 12$ **+1 pt**

$\displaystyle f_{ave} = \int_1^{12} \frac{1}{u}\,du$ **+1 pt**

$= \ln u\,\Big|_1^{12}$ **+1 pt**

$f_{ave} = \ln 12$ **+1 pt**

bonus problem Evaluate the integral $\displaystyle\int_{-4}^{4}\sqrt{16-x^2}\,dx$.

Solution: The integral is the area of the semicircle of radius 4. **+3 pts**

Therefore, $\displaystyle\int_{-4}^{4}\sqrt{16-x^2}\,dx = \frac{1}{2}\pi\cdot 4^2 = 8\pi.$ **+2 pts**