SAMPLE PROBLEMS

1.1. Find the vertical and horizontal asymptotes of the function

$$f(x) = \frac{-2x^2 - 3x + 5}{x^2 - 4x + 3}.$$

1.2. Find numbers a, b such that the function

$$f(x) = \begin{cases} ax^2 - 2x + 2 & \text{if } x \leq 3, \\ \dfrac{5\sin(x-3)}{x-3} & \text{if } 3 < x < 3 + \pi \\ b\cos(x-3) & \text{if } x \geq 3 + \pi, \end{cases}$$

is continuous. Justify your answer.

2.1. Differentiate:

a) $f(x) = (x^5 \tan(\sqrt[3]{x}))^{-3}$.

b) $g(x) = \dfrac{(\sin(e^x))^2}{e^x + x^e}$.

c) $h(t) = (\ln x)^{\sin^{-1}(x^2)}$

2.2. By implicit differentiation, find the equation of the tangent line to the curve

$$y\sin(x^2) - x^2\sin(y) = 0$$

at the point $(\sqrt{\pi}, \pi)$ on the curve.

3.1. A radioactive material loses one third of its mass after every 10 hours. How long is its half-life?

3.2. Find the following limits:

a) $\lim\limits_{x\to 1} \dfrac{x\ln x - x + 1}{(x-1)^2}$

b) $\lim\limits_{x\to 1} \dfrac{(\ln x)^2}{x^2 - 2x}$

c) $\lim\limits_{x\to 1} \dfrac{7^x - 7}{x - 1}$.

d) $\lim_{x\to 1^-} \dfrac{\cos^{-1}(x)}{\sqrt{1-x^2}}$.

4.1. Find the critical numbers of the function $f(x) = x^{1/5}(x-1)^2$ and determine whether each critical number is a local minimum, local maximum or neither of the two. Through applying calculus methods and drawing the sign chart, find all the intervals on which f is decreasing or increasing. Also determine the intervals on which the graph of f is concave up or concave down and find the inflection points of f.

4.2. If $4800cm^2$ of material is available to to make a box with a **square base** and an **open top**, find the dimensions of the box with the largest possible volume.

5.1. Find the total area between the x-axis and the graph of the given functions over the interval $[-1, 1]$:

a) $f(x) = 3\,x^2\sqrt[5]{(x^3+3)^2}$

b) $g(x) = |\sin x|$

5.2. Use the Fundamental Theorem of Caculus to find the derivative of the following functions:

a) $\displaystyle\int_1^{tanx} (t-t^7)^{10}\mathrm{d}t$

b) $f(x) = \displaystyle\int_x^{3x} \sin(t^2)\mathrm{d}t$

6.1. Evaluate the integrals:

(a) $\displaystyle\int x^2 \ln x \,\mathrm{d}x$

(b) $\displaystyle\int \tan^{-1}(x)\,\mathrm{d}x$

6.2. Evaluate the integrals:

(c) $\displaystyle\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} x^3\cos(x^2)\,\mathrm{d}x$

(d) $\displaystyle\int_0^{\pi/4} (\sin^2 t)\tan^3 t\,\mathrm{d}t$