Midterm Exam 1

Fall 2013 **Your name:** _____

No calculators, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers.

1. [15 pts] Evaluate the integral $\quad I = \displaystyle\int_3^5 \frac{5x}{x^2 + x - 6} \, dx.$ $\quad$ Simplify your answer.

2. [15 pts] Determine whether the integral $I = \int\limits_{2}^{5} \dfrac{5x}{x^2 + x - 6} \, dx$ is convergent or divergent.

Then evaluate the integral if it is convergent. You may use results obtained in the problem 1.

3. [15 pts] Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the curves $y = \sqrt{x}$ and $y = x^2$ about the y-axis.

4. [15 pts] Find the exact length of the curve $y = x^{3/2} - 2$ when $0 \leq x \leq 5$. Represent your answer as a single fraction.

5. [15 pts] A vertical plate is submerged in water and has the shape of a triangle with base of 2 feet and hight of 2 feet (see the picture below). Find the hydrostatic force in pounds against one side. The weight density of water is $62.5 = 125/2$ lb/ft^3. Represent the answer as a simple fraction.

water level

2 ft

2 ft
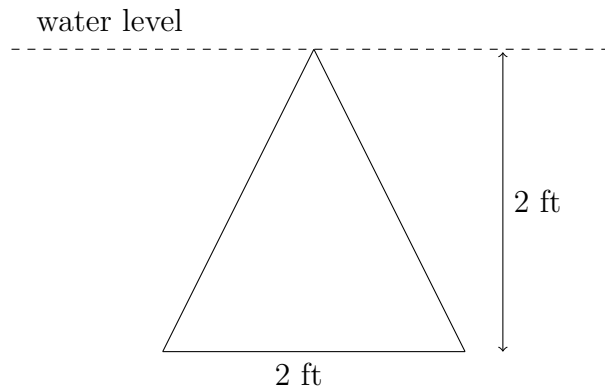
6. [10 pts] Given: $\bar{a} = \langle 3, \sqrt{3}, 2 \rangle$, $\bar{b} = \langle 5, -\sqrt{3}, z \rangle$, $\text{comp}_{\bar{a}} \bar{b} = 5$. Find z.

7. [15 pts] Find an equation of the plane that contains the line $x = 3 + 2t$, $y = t$, $z = 8 - t$ and is parallel to the plane $2x + 4y + 8z = 17$.

bonus problem [15 pts extra] A region in the shape of a triangle is bounded by the lines $y = 2x - 1$, $y = x$, and $y = -x + 5$. Find the volume of the solid obtained by rotating the region about the line $y = x$. Use any method.