Math 0230 **Midterm Exam 2** **Lecture time: 1 pm**

Fall 2013 **Your name:** _____

No calculators, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers.

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1. A curve C is defined by the parametric equations $x = t^2$, $y = t^3 - 3t$. Show that C has two tangents at the point $(3, 0)$ and find their equations.

Represent the equations in the slope-intercept form.

2. [10 points] Find the area of the region that lies inside the curve $r = \cos\theta$ and outside the curve $r = 1 - \cos\theta$.

3. [10 points] (a) [5 points] The point $(-2\sqrt{3}, -2)$ is given in the Cartesian coordinates. Find the polar coordinates of the point where $r \geq 0$ and $0 \leq \theta < 2\pi$.

.....

(b) [5 points] The point $\left(2, \frac{4\pi}{3}\right)$ is given in the polar coordinates. Find the Cartesian coordinates of the point.

4. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

(a) [5 points] $\displaystyle\sum_{n=1}^{\infty} \sqrt[n]{1.1}$

.....

(b) [5 points] $\displaystyle\sum_{n=1}^{\infty} (\sin 2)^n$

5. Determine whether the series is convergent or divergent. Do not find its sum. Show all your work. Mention a method used.

(a) [10 points] $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2 + \sqrt{n}}$

.....

(b) [10 points] $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{7n}$

.....

(c) [10 points] $\displaystyle\sum_{n=1}^{\infty} \frac{5^n(n-1)}{n!}$

6. Express the function as a power series. Find the radius of convergence R of the obtained power series.

(a) [10 points] $\dfrac{5}{5+x}$

.....

(b) [10 points] $\dfrac{5}{(5+x)^2}$

7. [10 points] Find the Taylor series for the function $f(x) = e^{2x}$ centered at $a = 1$. Represent the series in sigma $\sum$ form.

bonus problem [15 points extra] How accurate is the approximation

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^3}{24}$$

when $0 \leq x \leq \frac{1}{2}$? Simplify your answer and leave it in exact form.