Math 0230 **Midterm Exam 2** **Lecture time: 9 am**

Fall 2013 **Your name:** ______________________

No calculators, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers.

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1. [10 points] Find an equation of the tangent line to the parametric curve $x = 3t^2 + 1$, $y = 2t^3 + 1$ at the point where $t = -1$. Represent the equation in the slope-intercept form.

2. (a) [5 points] The point $(-2\sqrt{3}, -2)$ is given in the Cartesian coordinates. Find the polar coordinates of the point where $r \geq 0$ and $0 \leq \theta < 2\pi$.

(b) [5 points] The point $\left(2, \frac{4\pi}{3}\right)$ is given in the polar coordinates. Find the Cartesian coordinates of the point.

3. [10 points] Find the area of the region that lies inside the curve $r = \cos\theta$ and outside the curve $r = 1 - \cos\theta$.

4. Determine whether the series is convergent or divergent. If it is convergent, find its sum.

(a) [5 points] $\sum_{n=1}^{\infty} \sqrt[n]{2.5}$

. . . .

(b) [5 points] $\sum_{n=1}^{\infty} (\cos 1)^n$

5. Determine whether the series is convergent or divergent. Do not find its sum. Show all your work. Mention a method used.

(a) [10 points] $\sum_{n=1}^{\infty} \frac{1}{n^2 + 2n}$

(b) [10 points] $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$

(c) [10 points] $\sum_{n=1}^{\infty} \frac{3^n(n+2)}{n!}$

6. Express the function as a power series. Find the radius of convergence R of the obtained power series.

(a) [10 points] $\dfrac{2}{2+x}$

(b) [10 points] $\dfrac{2}{(2+x)^2}$

7. [10 points] Find the Taylor series for the function $f(x) = \ln x$ centered at $a = 1$. Represent the series in sigma $\sum$ form.

bonus problem [15 points extra] Find the sum of the series

$$3 - \frac{\pi^2}{3 \cdot 2!} + \frac{\pi^4}{27 \cdot 4!} - \frac{\pi^6}{243 \cdot 6!} + \cdots$$