Math 0230 **Quiz 1**

Fall 2013 **S o l u t i o n s**

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1. [4 points] Evaluate the integral $I = \int_0^1 \frac{x+5}{x^2+4x+3}\,dx$. Simplify your answer.

Solution: Partial fraction decomposition:

$$\frac{x+5}{x^2+4x+3} = \frac{1\cdot x+5}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3} = \frac{(A+B)x+(3A+B)}{(x+1)(x+3)}$$

A comparison of like terms gives: $A+B=1,\ 3A+B=5 \ \Rightarrow\ A=2,\ B=-1.$

Hence, $\frac{x+5}{x^2+4x+3} = \frac{2}{x+1} - \frac{1}{x+3}$ and

$$I = 2\int_0^1 \frac{dx}{x+1} - \int_0^1 \frac{dx}{x+3} = \Big[2\ln|x+1| - \ln|x+3|\Big]_0^1 = 2\ln 2 - \ln 4 - 2\ln 1 + \ln 3 = \ln 3$$

2. [4 points] Use Simpson's Rule to approximate the integral $I = \int_{-1}^3 \frac{x^3}{x^2+4}\,dx$ if $n=4$. Write your answer as a single rational number.

Solution: $\Delta x = \frac{3-(-1)}{4} = 1$. Then $I \approx \frac{1}{3}[f(-1)+4f(0)+2f(1)+4f(2)+f(3)]$

where $f(x) = \frac{x^3}{x^2+4}$, $f(-1) = -\frac{1}{5}$, $f(0)=0$, $f(1)=\frac{1}{5}$, $f(2)=1$, $f(3)=\frac{27}{13}$. Therefore

$$I \approx \frac{1}{3}\left[-\frac{1}{5}+0+\frac{2}{5}+4+\frac{27}{13}\right] = \frac{1}{3}\left[\frac{1}{5}+4+\frac{27}{13}\right] = \frac{1}{3}\cdot\frac{13+260+135}{65} = \frac{1}{3}\cdot\frac{408}{65} = \frac{408}{195}$$

3. [3 points] Evaluate the integral $I = \int_{-\pi/3}^{\pi/3} \tan^7 t\sec^4 t\,dt$. Write your answer as a single number.

Solution: The interval $[-\pi/3, \pi/3]$ is symmetric and the function $f(t) = \tan^7 t\sec^4 t$ is continuous on the interval and odd:

$f(-t) = \tan^7(-t)\sec^4(-t) = (-\tan t)^7\sec^4 t = -\tan^7 t\sec^4 t = -f(t).$

Hence $I = 0$.

bonus problem [5 points extra] Evaluate the integral $I = \int\limits_1^4 \frac{\sqrt{x}}{\sqrt{x}+2}\, dx.$ Simplify your answer.

Solution: u-sub: $u = \sqrt{x} + 2,\ \sqrt{x} = u - 2,\ du = \frac{dx}{2\sqrt{x}},\ dx = 2\sqrt{x}du = 2(u-2)du,$

$u(1) = 3,\ u(4) = 4.$

$$I = \int\limits_3^4 \frac{2(u-2)^2}{u}\, du = \int\limits_3^4 (2u - 8 + 8u^{-1})\, du = \Big[u^2 - 8u + 8\ln|u| \ \Big]_3^4$$

$$= 16 - 32 + 8\ln 4 - 9 + 24 - 8\ln 3 = 8\ln\frac{4}{3} - 1$$