Math 0230 **Quiz 4**

Fall 2013 **S o l u t i o n s**

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1. [5 points] Find $\lim_{n\to\infty} \frac{\sin n}{n+2}$ by applying the Squeeze theorem. Support your solution.

No credit will be given if the Squeeze theorem is not used.

Solution: $\lim_{n\to\infty} \left|\frac{\sin n}{n+2}\right| = \lim_{n\to\infty} \frac{|\sin n|}{n+2}$

$0 < \frac{|\sin n|}{n+2} < \frac{1}{n+2} < \frac{1}{n}$ when $n \geq 1$. $\lim_{n\to\infty} 0 = 0$, $\lim_{n\to\infty} \frac{1}{n} = 0$.

Then by the Squeeze theorem $\lim_{n\to\infty} \frac{|\sin n|}{n+2} = 0$ and then $\lim_{n\to\infty} \frac{\sin n}{n+2} = 0$.

2. [5 points] Using the Comparison Test determine whether the series $\sum_{n=1}^{\infty} \frac{\sin^2 n + 1}{(n+2)^2}$ is convergent or divergent.

Solution: $0 \leq \frac{\sin^2 n + 1}{(n+2)^2} \leq \frac{2}{(n+2)^2} \leq 2 \cdot \frac{1}{n^2}$. The last term represents two times the p-series with $p = 2$ which is convergent. By the Comparison Test the series $\sum_{n=1}^{\infty} \frac{\sin^2 n + 1}{(n+2)^2}$ is also convergent.

bonus problem [5 points extra] For the Fibonacci sequence ($f_1 = f_2 = 1$, $f_n = f_{n-1} + f_{n-2}$) show that $\frac{1}{f_{n-1}f_{n+1}} = \frac{1}{f_{n-1}f_n} - \frac{1}{f_n f_{n+1}}$ and find the series $\sum_{n=2}^{\infty} \frac{1}{f_{n-1}f_{n+1}}$.

Solution: $\frac{1}{f_{n-1}f_n} - \frac{1}{f_n f_{n+1}} = \frac{f_n f_{n+1} - f_{n-1}f_n}{f_{n-1}f_n^2 f_{n+1}} = \frac{f_n(f_{n-1} + f_n) - f_{n-1}f_n}{f_{n-1}f_n^2 f_{n+1}}$

$= \frac{f_n^2}{f_{n-1}f_n^2 f_{n+1}} = \frac{1}{f_{n-1}f_{n+1}}$

$$\sum_{n=2}^{\infty} \frac{1}{f_{n-1}f_{n+1}} = \sum_{n=2}^{\infty} \left(\frac{1}{f_{n-1}f_n} - \frac{1}{f_n f_{n+1}}\right)$$

$$= \left(\frac{1}{f_1 f_2} - \frac{1}{f_2 f_3}\right) + \left(\frac{1}{f_2 f_3} - \frac{1}{f_3 f_4}\right) + \left(\frac{1}{f_3 f_4} - \frac{1}{f_4 f_5}\right) + \cdots = \frac{1}{f_1 f_2} = 1.$$