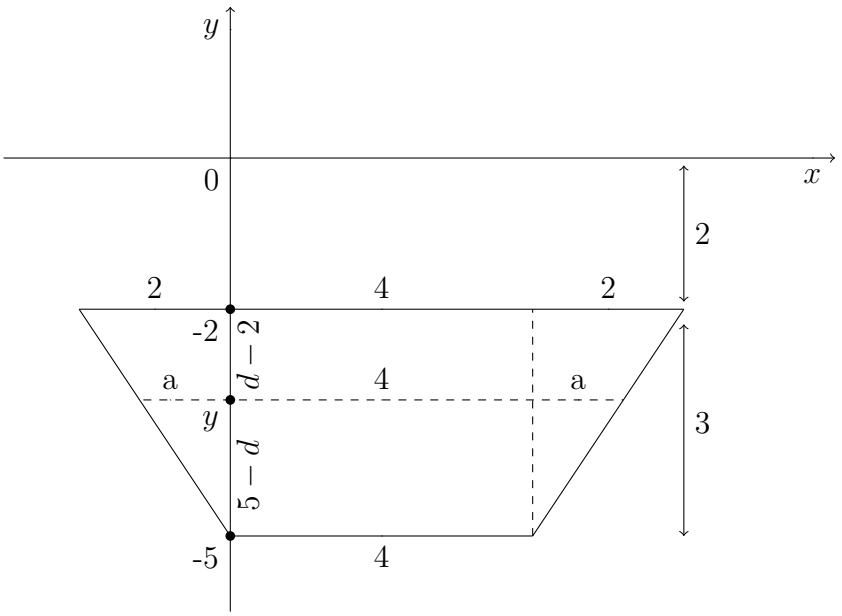
Example 3. A plate in an irrigation canal is in the form of a trapezoid 4 feet wide at the bottom, 8 feet wide at the top, with the height equal to 3 feet. It is placed vertically in the canal and is submerged in water 2 feet deep. Find the hydrostatic force in pounds on the plate. The weight density of water is $62.5 = 125/2$ lb/ft^3.

Solution 1: Let's place the coordinate axes as it is shown on the picture. We use horizontal strips of equal width δy and consider a strip on the depth $d = -y$ with $-5 \le y \le -2$ according to the picture.



By similar triangles: $\dfrac{a}{2} = \dfrac{5-d}{3} = \dfrac{5+y}{3}$, $a = \dfrac{2}{3}(5+y)$.

The length of the strip is $l = 4 + 2a = 4 + \dfrac{4}{3}(5+y) = \dfrac{4}{3}(8+y)$ and its area is $\Delta A = \dfrac{4}{3}(8+y)\,\Delta y$.

The pressure on the strip is $P = 62.5d = -62.5y$ (note that $d = -y$ since y is negative) and the force is

$$\Delta F = P\Delta A = -\frac{125}{2}y \cdot \frac{4}{3}(8+y)\,\Delta y = -\frac{250}{3}(8y + y^2).$$

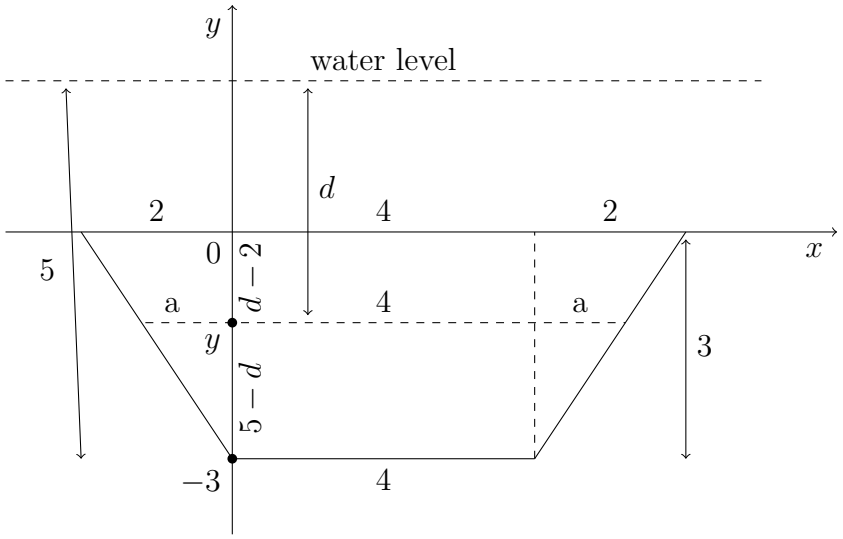
Then the total force is

$$F = -\int_{-5}^{-2} \frac{250}{3}(8y + y^2)\,dy = -\frac{250}{3}\left[4y^2 + \frac{y^3}{3}\right]_{-5}^{-2} = -\frac{250}{3}\left[16 - \frac{8}{3} - 100 + \frac{125}{3}\right]$$

$$= -\frac{250}{3}\left[-\frac{135}{3}\right] = 250 \cdot 15 = 3750 \text{ lb}$$

Solution 2: Now we place the coordinate axes differently. See the picture below. As before we use horizontal strips of equal width δy and consider a strip on the depth $d = 2 - y$ with

$-3 \le y \le 0$ according to the picture.



By similar triangles: $\dfrac{a}{2} = \dfrac{5-d}{3} = \dfrac{5-2+y}{3}$, $a = \dfrac{2}{3}(3+y)$.

The length of the strip is $l = 4 + 2a = 4 + \dfrac{4}{3}(3+y) = \dfrac{4}{3}(6+y)$ and its area is $\Delta A = \dfrac{4}{3}(6+y)\,\Delta y$.

The pressure on the strip is $P = 62.5d = 62.5(2-y)$ and the force is

$$\Delta F = P\Delta A = \frac{125}{2}(2-y) \cdot \frac{4}{3}(6+y)\,\Delta y = \frac{250}{3}(12 - 4y - y^2).$$

Then the total force is

$$F = \int_{-3}^{0} \frac{250}{3}(12 - 4y - y^2)\,dy = \frac{250}{3}\left[12y - 2y^2 - \frac{y^3}{3}\right]_{-3}^{0} = \frac{250}{3}\left[0 - (-36 - 18 + 9)\right]$$

$$= \frac{250}{3} \cdot 45 = 250 \cdot 15 = 3750 \text{ lb}$$