Math 0230 Calculus 2 Lectures

Chapter 10 Vectors and the Geometry of Space

Numeration of sections corresponds to the text

James Stewart, Essential Calculus, Early Transcendentals, Second edition.

Section 10.1 Three-Dimensional Coordinate Systems

3d coordinate system, xyz-space. x-axis, y-axis, and z-axis. The right hand rule. Any point has three coordinates. Three coordinate planes: xy, xz, and yz. They divide the space into eight octants. Projections of a point onto coordinate planes.

One equation connecting x, y, and z (some variables can be missed) defines a 2d surface in 3d space (3 dimensions - one relation = 2d). If the relation (equation) is linear, then the surface is a plane.

Example 1. (a) $z = 2$ is a linear equation in 3d space. Hence, it defines a plane.

(b) $2x + y - z - 2 = 0$ is a linear equation in 3d space. Hence, it defines a plane.

(c) $y = x$ is a linear equation in 3d space. Hence, it defines a plane. [Notice that it is a line in 2d xy plane]

Distance Formula in 3d

The distance $|P_1P_2|$ between points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

Here is an example of the well known nonlinear surface:

Equation of a Sphere

An equation of a sphere with center $C(h, k, l)$ and radius r is

$$(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2$$

In particular, if the center is the origin $O(0, 0, 0)$, then an equation of the sphere is

$$x^2 + y^2 + z^2 = r^2$$

a Ball

A ball with center $C(h, k, l)$ and radius r is described by an ineqality

$$(x-h)^2 + (y-k)^2 + (z-l)^2 \leq r^2$$

Example 2. (a) Which of the points $P(2, 3, -1)$, $Q(-4, 2, 0)$, $R(3, -5, 1)$ are closest to the origin? To the xz-plane. What are their projections on the coordinate planes?

(b) Find distances between the points.

Example 3. Show that the equation

$$x^2 + y^2 + z^2 = 4x - 2y$$

represents a sphere and find its center and radius

Solution: $x^2 - 4x + y^2 + 2y + z^2 = 0$, $x^2 - 4x + 4 + y^2 + 2y + 1 + z^2 = 4 + 1$

and complete squares: $(x-2)^2 + (y+1)^2 + z^2 = (\sqrt{5})^2$

The center is at $(2, 1, 0)$, the radius is $\sqrt{5}$.

Section 10.2 Vectors

Section 10.3 The Dot Product

Section 10.4 Equations of Lines and Planes