Midterm Exam 2

Spring 2012 Math 0280

100 points total **Your name:** _____

No calculators. Show all your work (no work = no credit). Write neatly.

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1. (15 points) Prove that every line through the origin in $\mathbb{R}^2$ is a subspace of $\mathbb{R}^2$.

2. (15 points) Prove that T is a linear transformation if

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -y \\ x + 2y \\ 3x - 4y \end{bmatrix}.$$

You can apply any method.

3. (20 points) By calculating $\det(A-\lambda I)$ find all the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$$

and an eigenvector that corresponds to the larger eigenvalue.

4. For the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

(a) (10 points) find a basis for $col(A)$

(b) (10 points) find a basis for $null(A)$

5. (15 points) Find the standard matrix of the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ which is the projection onto the line $y = 2x$.

6. (15 points) Find a basis for the span of the given vectors from among the vectors themselves

$$\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

bonus problem (10 points extra) Find the standard matrix of the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ which is the reflection in the line $y = mx$.