Quiz 4

Spring 2012

Math 0280

Your name: Solutions

1. [8 points] The transformation T stretches a vector by a factor of 2 in the x-component and by a factor of 3 in the y-component. Show that the transformation T from $\mathbb{R}^2$ to $\mathbb{R}^2$ is linear by showing that it is a matrix transformation.

Let $\bar{x} = \begin{bmatrix} x \\ y \end{bmatrix}$, then

$$T\bar{x} = T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 3y \end{bmatrix}.$$

$$\bar{e}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad T\bar{e}_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \quad \bar{e}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad T\bar{e}_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Hence $T_A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ (matrix of T)

Check: $T_A\bar{x} = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x \\ 3y \end{bmatrix} = T\bar{x}$

Hence T is linear.

2. [7 points] Using the definition of a linear transformation show that

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x - y + z \\ 2x + y - 3z \end{bmatrix}$$

is a linear transformation.

Let $\bar{u} = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$, $\bar{v} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$

$$T(\bar{u}+\bar{v}) = T\begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{bmatrix} = \begin{bmatrix} (x_1+x_2)-(y_1+y_2)+(z_1+z_2) \\ 2(x_1+x_2)+(y_1+y_2)-3(z_1+z_2) \end{bmatrix}$$

$$T\bar{u} + T\bar{v} = \begin{bmatrix} x_1 - y_1 + z_1 \\ 2x_1 + y_1 - 3z_1 \end{bmatrix} + \begin{bmatrix} x_2 - y_2 + z_2 \\ 2x_2 + y_2 - 3z_2 \end{bmatrix} =$$

$$= \begin{bmatrix} x_1 + x_2 - y_1 - y_2 + z_1 + z_2 \\ 2x_1 + 2x_2 + y_1 + y_2 - 3z_1 - 3z_2 \end{bmatrix} = \begin{bmatrix} x_1+x_2-(y_1+y_2)+z_1+z_2 \\ 2(x_1+x_2)+(y_1+y_2)-3(z_1+z_2) \end{bmatrix}$$

Hence $T(\bar{u}+\bar{v}) = T\bar{u} + T\bar{v}$

$$T(c\bar{u}) = T\begin{bmatrix} cx_1 \\ cy_1 \\ cz_1 \end{bmatrix} = \begin{bmatrix} cx_1 - cy_1 + cz_1 \\ 2cx_1 + cy_1 - 3cz_1 \end{bmatrix} = c\begin{bmatrix} x_1 - y_1 + z_1 \\ 2x_1 + y_1 - 3z_1 \end{bmatrix}$$

$$= cT\bar{u}$$

Therefore T is a lin. tr'n by the def'n