Math 0290 **Midterm Exam 2**

Fall 2014 **Your name:** _____

No calculators, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers when possible.

For each problem (except the bonus) you will get 40% of the maximum points if you do not write a solution. If you write a solution then your score will vary from 0 to maximum.

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1. (15 points) Find the inverse Laplace transform of the function $F(s) = \dfrac{1}{s^2 - s}$, $s \geq 1$.

2. (15 points) Solve the initial-value problem $y' - y = g(t),\ y(0) = 0,$ where

$$g(t) = \begin{cases} 0, & \text{for } 0 \le t < 1 \\ 1, & \text{for } 1 \le t < 2 \\ 0, & \text{for } t \ge 2 \end{cases}$$

Create a piecewise definition for your solution that doesn't use the Heaviside function. Show all your work. You may use results from the previous problem.

3. (15 points) Using the unit impulse response function and convolution find the solution to the initial-value problem

$$y'' + 9y = g(t), \qquad y(0) = 1, \quad y'(0) = 0,$$

where $g(t)$ is a piecewise continuous function.

4. (15 points) For the initial-value problem $y' = t(2y + t), \quad y(0) = 1$ calculate the second iteration y_2 of Euler's method with step size $h = 0.1$.

5. (15 points) Consider the initial value problem

$$y'' - 5y' + 2y = 2t^3, \qquad y(0) = 2, \quad y'(0) = 1$$

(a) (15 points) Write the IVP as a system of first order equations.

(b) (15 points) Write the obtained system in vector form. Don't use matrices. Define all vectors.

6. For the system of differential equations

$$x' = 4x - 2x^2 - xy$$
$$y' = 4y - xy - 2y^2$$

(a) (15 points) Find x-nullcline and y-nullcline.

(b) (15 points) Plot x-nullcline and y-nullcline. Use bold lines for x-nullcline and dashed lines for y-nullcline. Plot the equilibrium points. Don't calculate coordinates of equilibrium points, just plot them.

bonus problem (15 points extra) Find the inverse Laplace transform of the function $F(s) = \dfrac{s^2 - 4}{(s^2 + 4)^2}$