Math 0290 **Midterm Exam 2**

Spring 2014 **Your name:** _____

No calculators, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers.

For each problem (except the bonus) you will get 40% of the maximum points if you do not write a solution. If you write a solution then your score will vary from 0 to maximum.

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1. (15 points) Use the Laplace transform to solve the initial-value problem

$$y'' + 4y' + 4y = 0 \qquad y(0) = 3, \quad y'(0) = -5.$$

2. (15 points) Find the unit impulse response to the equation $e'' + 6e' + 10e = \delta(t)$.

3. (15 points) Find the solution to the initial-value problem

$$y'' + 6y' + 10y = g(t), \qquad y(0) = 0, \quad y'(0) = 5,$$

where $g(t)$ is a piecewise continuous function.

4. (15 points) For the initial-value problem

$$y' = 3t - y, \quad y(0) = 2$$

calculate the first two iterations of Euler's method with step size $h = 0.2$.

5. For the system of differential equations

$x' = (y - x)(y + x^2 - 2)$

$y' = (y + x)(y - 2)$

(a) (5 points) find x-nullclines and y-nullclines.

(b) (5 points) calculate the coordinates of the equilibrium points.

(c) (10 points) Plot x-nullclines and y-nullclines. Use solid lines for x-nullclines and dashed lines for y-nullclines. Plot the equilibrium points and label them with their coordinates.

6. (20 points) Find the general solution to the system $y' = Ay$, where $A = \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$. Write the answer as a single vector.

bonus problem (15 points extra) Find real valued solutions $y_1(t)$ and $y_2(t)$ of the system

$$\begin{aligned} y_1' &= y_1 + 2y_2 \\ y_2' &= -5y_1 + 3y_2 \end{aligned}$$