Math 0290 **Midterm Exam 1**

Fall 2015 **Name:** _____

No calculators, no books. Show all your work (no work = no credit).
Write neatly. Simplify your answers when possible.

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1. (10 points) A 0.25 kg mass is attached to a spring having a spring constant 9 kg/s^2. The system is displaced 0.3 m from its equilibrium position and released from rest. If there is no dumping present, find the amplitude, frequency, and phase angle of the resulting motion.

2. Solve the initial-value problem. Find the interval of existence of the solution. Show all the work. Mention type of the given differential equation.

(a) (15 points) $y \ln y + x\, y' = 0, \qquad y(1) = e^{-2}, \quad$ where $y' = \dfrac{dy}{dx}$.

(b) (15 points) $(x^2 + 1)y' + 2xy = 6x, \qquad y(0) = -1.$

3. Consider the equation $y'' + 3y' - 4y = 0$.

(a) (10 points) Show that $y_1 = e^t$ and $y_2 = e^{-4t}$ are solutions of the given equation.

(b) (10 points) Use Wronskian to show that y_1 and y_2 are linearly independent and hence form the fundamental set of solutions.

(c) (10 points) Find a solution of the given differential equation satisfying the initial conditions $y(0) = 3,\ y'(0) = -2$.

4. In the previous problems it was proven that $y_1 = e^t$, $y_2 = e^{-4t}$ form the fundamental set of solutions of the homogeneous equation $y'' + 3y' - 4y = 0$. Use this result to find a general solution of the equation

$$y'' + 3y' - 4y = 10e^t$$

(a) (15 points) by using the method of undetermined coefficients.

(b) (15 points) by using the method of variation of parameters. You may use the Wronskian found before and formulas for evaluation v_1 and v_2 directly.

bonus problem (15 points extra) Find the general solution of the equation $ty'' = y'$.