Math 0290   **Midterm Exam 2**

No calculators, no books. Show all your work (no work = no credit). Write neatly. Simplify your answers when possible.

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1. (15 points)   Find the inverse Laplace transform of the function $F(s) = \dfrac{6}{s^2 - 3s}, \; s \geq 3.$

2. (15 points) By using Laplace transform solve the initial-value problem $y' - 3y = g(t)$, $y(0) = 0$, where

$$g(t) = \begin{cases} 0, & 0 \le t < 4 \\ 6, & t \ge 4 \end{cases}$$

Create a piecewise definition for your solution that does not use the Heaviside function.

Show all your work. You may use results from the previous problem.

3. (15 points)   Using the unit impulse response function and convolution find the solution to the initial-value problem

$$y'' + 25y = g(t), \qquad y(0) = 1, \quad y'(0) = 5,$$

where $g(t)$ is a piecewise continuous function.

4. (15 points) For the initial-value problem $\quad y' = 3t(y + t), \quad y(0) = 2$

calculate the second iteration y_2 of Euler's method with step size $h = 0.1$.

5. For the system of differential equations

$$x' = x - xy$$
$$y' = x^2y - y^2$$

(a) (15 points)   find x-nullcline and y-nullcline. Draw a plot.

(b) (10 points)   find equilibrium points. Mark them on the plot.

6. (15 points) Find the general solution to the system $\bar{y}' = A\bar{y}$, where $A = \begin{pmatrix} -1 & 6 \\ -3 & 8 \end{pmatrix}$.

Write the answer as a single vector.

bonus problem (15 points extra) Find the inverse Laplace transform of the function $F(s) = \dfrac{s^2}{(s^2 + 1)^2}$.
Simplify your answer.