Math 0290   **Midterm Exam 1**

Fall 2016   **Name:** _____

No calculators, no books. Show all your work (no work = no credit).

Write neatly. Simplify your answers when possible.

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1. (a) (15 points)   Solve the initial value problem $x' = 1 + x \tan t, \qquad x(0) = 5$

(b) (5 points)   Find the interval of existence.

2. (15 points) Solve the initial value problem $y' = \dfrac{\sin x}{2y}$, $y(\pi) = -2$.

3. (15 points) Consider a simple RLC circuit with $R = 2\,\Omega$, $C = \frac{1}{4}$ F, $E = 12$ V and there is no inductor. Find $I(t)$ if $I(0) = 2$ A.

4. (15 points)   For the initial-value problem

$$y' = \frac{y}{t}, \qquad y(1) = 2$$

find an approximate value of $y(1.2)$. Use Euler's method with the step size $h = 0.1$. Simplify your answer.

5.   A 2-kg mass when attached to a spring, stretches the spring to a distance of 4.9 m.

(a) (5 points)   Calculate the spring constant.

(b) (10 points)   The system is placed in a viscous medium that supplies a damping constant $\mu = 6$ kg/s. The system is allowed to come to rest. Then the mass is displaced 2 m in the downward direction and given a sharp tap, imparting an instantaneous velocity of 1 m/s in the downward direction. Find the position of the mass as a function of time.

6. For the equation $y'' - 4y' - 5y = 12e^{-t}$.

(a) (5 points) Find the fundamental set of solutions of the corresponding homogeneous equation.

(b) (10 points) Find a particular solution by using the method of undetermined coefficients.

(c) (5 points) Find the general solution.

bonus problem (15 points extra)   Suppose $y(x)$ is a differentiable nonnegative function with $y(0) = 0$. Find $y(x)$ if the area under the curve $y = y(x)$ from 0 to x $(x > 0)$ is always equal to one fourth the area of the rectangle with vertices at $(0, 0)$ and $(x, y(x))$.