Math 0290 **Midterm Exam 2**

Fall 2016 **Name:** _____

No calculators, no books. Show all your work (no work = no credit).
Write neatly. Simplify your answers when possible.

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1. (15 points) By using Laplace transform solve the initial-value problem

$$y'' + 4y = 4\cos 2t, \quad y(0) = 0, \quad y'(0) = 0$$

Do not use the convolution.

2. Consider the initial-value problem $y' + y = g(t),\ y(0) = 0,$ where

$$g(t) = \begin{cases} 0, & \text{for } 0 \le t < 3 \\ t, & \text{for } t \ge 3 \end{cases}$$

(a) (5 points) Describe the function $g(t)$ in terms of the Heaviside function.

(b) (10 points) By using Laplace transform solve the initial-value problem. Do not use the convolution. Hint:

$$\frac{3s^2+1}{s^2(s+1)} = -\frac{1}{s} + \frac{1}{s^2} + \frac{4}{s+1}$$

(c) (5 points) Create a piecewise definition for your solution $y(t)$ that does not contain the Heaviside function.

3. (20 points) Use Laplace transform and the convolution to find a solution to the initial-value problem

$$y'' + 4y = 2\sqrt{t}, \qquad y(0) = 1, \quad y'(0) = -6$$

4. For the homogeneous linear system
$$\begin{aligned} y_1' &= y_1 + 2y_2 \\ y_2' &= -y_1 + 4y_2 \end{aligned}$$

(a) (3 points) Find the corresponding 2×2 matrix A

(b) (5 points) Find eigenvalues of the matrix A

(c) (9 points) Find eigenvectors of the matrix A

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(d) (5 points) Find the general solution of the system

(e) (3 points) Find $y_1(t)$ and $y_2(t)$.

5. For the system of differential equations
$$x' = 2xy - x^3$$
$$y' = (y-2)(4y-3x)$$

(a) (13 points) find x-nullcline and y-nullcline. Draw a plot.

(b) (7 points) find equilibrium points. Clearly mark them on the plot.

bonus problem (10 points extra) Find the inverse Laplace transform of the function

$$G(s) = \frac{s-3}{(s^2-6s+10)^2}$$

Your score is

(out of 100)

Do not write above this line