Math 0290 **Midterm Exam 2**

Spring 2016 **Name:** _____

No calculators, no books. Show all your work (no work = no credit).
Write neatly. Simplify your answers when possible.

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1. (15 points) By using Laplace transform solve the initial-value problem $y' - y = 6e^{-2t},\ y(0) = 0$. Show all work.

2. (15 points) Use the Heaviside function to redefine the function

$$g(t) = \begin{cases} 1, & 0 \le t < 1 \\ \sin(2\pi t), & t \ge 1 \end{cases}$$

then find Laplace transform of $g(t)$.

3. (15 points) Using the unit impulse response function and convolution find the solution to the initial-value problem

$$y'' - 6y' + 13y = g(t), \qquad y(0) = 0, \quad y'(0) = 4,$$

where $g(t)$ is a piecewise continuous function.

4. (15 points) For the initial-value problem $y' = \dfrac{y}{t+1}, \quad y(0) = 1$

calculate the second iteration y_2 of Euler's method with step size $h = 0.1$. Simplify your answer.

5. For the system of differential equations

$$x' = x(6 - 2x - 3y)$$
$$y' = y(1 - x - y)$$

(a) (15 points) find x-nullcline and y-nullcline. Draw a plot.

(b) (10 points) find equilibrium points. Mark them on the plot.

6. (15 points) Find general solutions $y_1(t)$ and $y_2(t)$ of the system

$$y_1' = 3y_1 - y_2$$
$$y_2' = \ \ y_1 + y_2$$

bonus problem (15 points extra) Find all equilibrium points for the system of differential equations

$$x' = 1 - 2y$$
$$y' = x \sin x + xy$$