Math 0290 **Midterm Exam 1**

Spring 2017 **Name:** _____

No calculators, no books. Show all your work (no work = no credit).
Write neatly. Simplify your answers when possible.

---

1. A ball is dropped from the top of a cliff with the initial velocity 0 meters/second. The ball has mass of 0.1 kg. The air resistance force proportional to velocity, given that a resistance of 2 Newtons is experienced at a **downward** velocity of 4 m/sec.

   (a) (10 points) Write down a differential equation that describes the ball motion and solve it.

   (b) (5 points) How long will it take the ball to reach downward velocity 1.5 m/sec? Assume that the gravity constant is 10 m/sec^2. Leave your answer in exact form.

2. Consider the initial value problem: $\dfrac{dy}{dx} = \dfrac{6x+4}{3y^2}, \qquad y(0) = 1$

(a) (5 points) Find the explicit particular solution of this problem.

(b) (5 points) Determine the interval of existence of your particular solution.

(Note: If you do not manage to solve (a), then assume that the solution is $y(x) = \frac{1}{9}(x^2 + 7x + 10)$).

(c) (5 points) Use the Euler's method with step size $h = 1$ to derive an approximation to the particular solution at $x = 1$.

(d) (5 points) Determine the error of the Euler's method at $x = 1$.

3. (15 points) A tank is filled with 400 gallons of pure water. Brine containing 2 pounds of salt per gallon is pumped into the tank at a rate of 4 gal/min. The well-mixed solution is pumped out at a rate of 8 gal/min. Find the number of pounds of salt in the tank as a function of time.

4. (10 points) Consider a simple RLC circuit with $R = 5\ \Omega$, $C = 0.5$ F, $E = 10$ V and there is no inductor. If the charge on the capacitor is zero at time $t = 0$, find the ensuing charge on the capacitor and the current in the circuit at time t. Find $I(t)$ if $I(0) = 2$ A.

5. For the equation $y'' + y' - 2y = 9e^t - 6t$.

(a) (5 points) Find the fundamental set of solutions of the corresponding homogeneous equation.

(b) (10 points) Find a particular solution by using the method of undetermined coefficients.

(c) (10 points) Find a particular solution by using the method of variation of parameters.
[Hint: $\int te^{at}\,dt = \dfrac{at-1}{a^2}\,e^{at}\ \ (+C)$].

6. A forced mass spring system with an external driving force is modeled by

$$x'' + 2x' + 10x = 15 \sin 4t,$$

where t is measured in seconds and x in meters.

(a) (5 points) Find the transient state, i.e. the solution to the associated homogeneous equation.

(b) (5 points) Find the steady-state, i.e. a particular solution, to the forced equation. What are the amplitude and the phase of the steady-state?

(c) (5 points) Find the general solution.

bonus problem (10 points extra) Explain the difference in particular solutions obtained in parts (b) and (c) of the problem 5 and how it may affect the solution of the differential equation $y''+y'-2y = 9e^t-6t$.