Math 0290 **Midterm Exam 2**

Spring 2017 **Name:** _____

No calculators, no books. Show all your work (no work = no credit).
Write neatly. Simplify your answers when possible.

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1. Results of this problem will be used in other problems. Therefore do all calculations carefully and double check them.

   Find the inverse Laplace transform of the functions

   (a) (10 points) $Y(s) = \dfrac{2}{s^2 - 2s + 5}$

   (b) (10 points) $F(s) = \dfrac{s}{(s^2 + 9)^2}$

2. (10 points) By using Laplace transform solve the initial-value problem

$$y'' + 9y = \cos 3t, \qquad y(0) = 0, \quad y'(0) = 0.$$

Do not use the convolution. [Hint: use a result from the previous problem].

3. Consider the initial-value problem $y' + 3y = g(t),\ y(0) = 5,$ where

$$g(t) = \begin{cases} 0, & \text{for} \quad 0 \le t < 2 \\ 3t - 5, & \text{for} \quad t \ge 2 \end{cases}$$

(a) (4 points) Describe the function $g(t)$ in terms of the Heaviside function.

(b) (8 points) By using Laplace transform solve the initial-value problem. Do not use the convolution.

(c) (4 points) Create a piecewise definition for your solution $y(t)$ that does not contain the Heaviside function.

4. (12 points) Use Laplace transform and the convolution to find a solution to the initial-value problem

$$y'' - 2y' + 5y = t^{2/3}, \qquad y(0) = 0, \quad y'(0) = -2$$

[Hint: use a result from the problem 1].

5. For the system of differential equations
$$x' = -x + 4y$$
$$y' = -2x + 5y$$

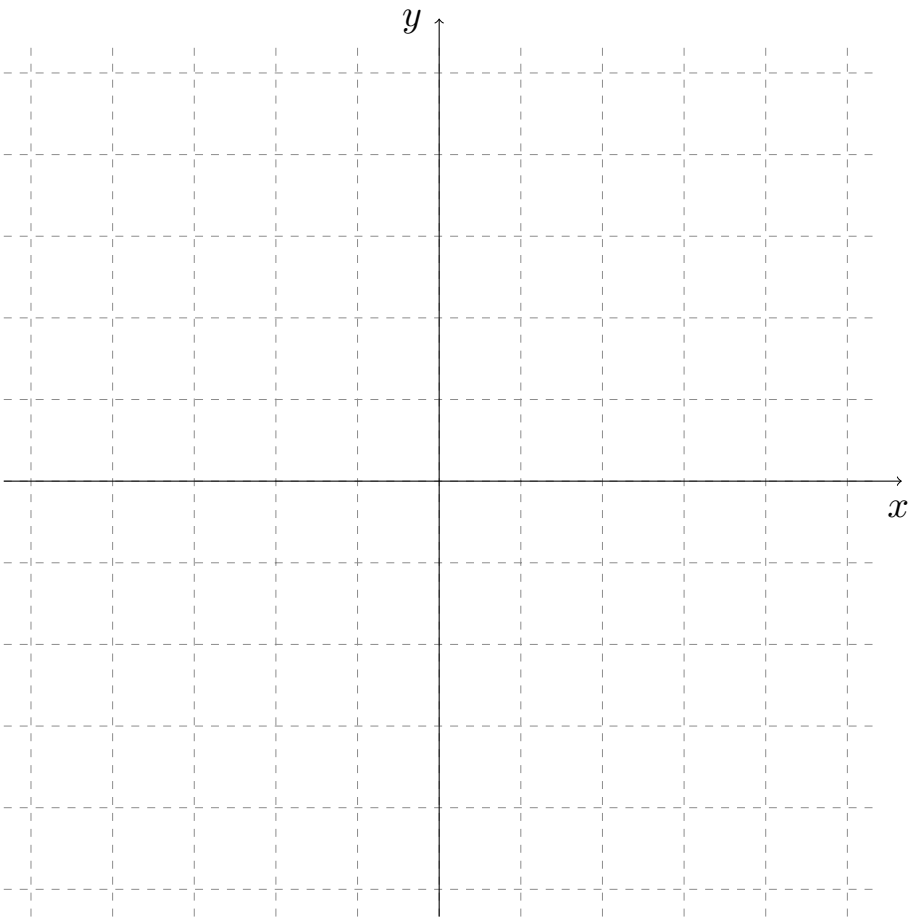
(a) (8 points) Find eigenvalues and eigenvectors.

(b) (5 points) Find the FSS and the general solution.

(c) (3 points) Find $x(t)$ and $y(t)$.

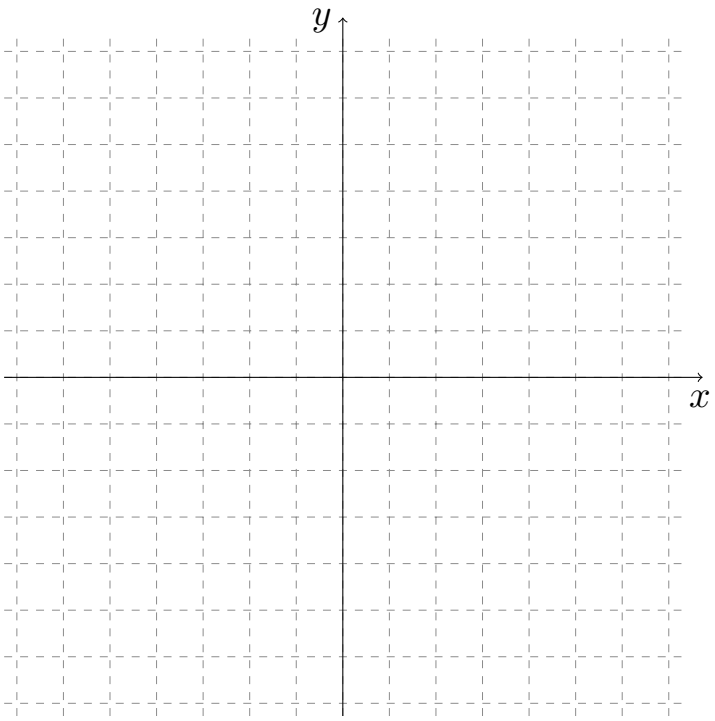
(d) (3 points) Determine the type of the equilibrium point.

(e) (6 points) Draw the phase plane portrait.



6. For the system of differential equations
$$x' = (x+2)(x-y^2)$$
$$y' = x^2 + y$$

(a) (8 points) Find x-nullcline and y-nullcline. Draw a plot.



(b) (3 points) Find equilibrium points and list them below. Clearly mark them on the plot.

Equilibrium points are:

(c) (6 points) Determine the type of the most right equilibrium point (saddle point, nodal sink, nodal source, spiral sink, or spiral source). If the type cannot be determined, explain why. Do not draw the phase portrait.

bonus problem (10 points) Find the Laplace transform of the periodic function $f(t)$ defined for $t \in [0, \infty)$ graph of which on the interval $t \in [0, 6)$ is given below