

Fall 2015

Name: _____

No calculators, no books. Show all your work (no work = no credit).

Write neatly. Simplify your answers when possible.

Part 1. 30 % of the test score.

Give the definition of

(a) (1 point) partition of a set

(b) (1 point) well ordering property of \mathbb{N}

(c) (1 point) countable set

(d) (1 point) least upper bound property of a set

Part 2. 70 % of the test score.

1. (5 points) Show that the proposition $(P \Rightarrow Q) \Leftrightarrow (\sim Q \Rightarrow \sim P)$ is a rule of inference.

2. (5 points) On the set $\{(a, b)\}$ of all ordered pairs of natural numbers, define $(a_1, b_1) \sim (a_2, b_2)$ if $a_1 b_2 = a_2 b_1$. Show that this defines an equivalence relation.

3. For each $n \in \mathbb{N}$ let $A_n = \{(-1)^n n + k : k \in \mathbb{N}\}$.

(a) (3 points) Find $\bigcap_{n=1}^{\infty} A_n$

(b) (2 points) Find $\bigcup_{n=1}^{\infty} A_n$

4. (5 points) Prove that 24 divides the number $5^{2n} - 1$ for any natural n .

5. (5 points) Prove that if $x \in \mathbb{R}$ such that $x \geq 0$ and $x < a$ for all $a \in \mathbb{R}$ where $a > 0$ then $x = 0$.