Math 0413 **Sample Midterm Exam**

Fall 2015 **Name:** _____

No calculators, no books. Show all your work (no work = no credit).
Write neatly. Simplify your answers when possible.

Part 1. 30 % of the test score.

Give the definition of

(a) (1 point) rule of inference

(b) (1 point) well ordering property of $\mathbb{N}$

(c) (1 point) inverse image of a function

(d) (1 point) ordered field

Part 2. 70 % of the test score.

1. (5 points) Show that the proposition $(P \Leftrightarrow Q) \wedge (\sim P \wedge Q)$ is a contradiction.

2. (5 points) For $(a, b), (c, d) \in \mathbb{R}^2$ define $(a, b) \sim (c, d)$ to mean that $2a - b = 2c - d$.
   Prove that $\sim$ is an equivalence relation on $\mathbb{R}^2$.

3. Let $f\colon\ A \to B$. Let C and D be subsets of B. Show that
$$f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$$

4. (5 points) Let F be an ordered field and $x, y \in F$. Prove that if $0 < x < y$ then $0 < 1/y < 1/x$.

5. (5 points) Prove that $\sqrt{5} \notin \mathbb{Q}$.