Math 0413 **Final Exam** Summer 2016

80 minutes **Name:** _____

No calculators, no books. Show all your work (no work = no credit).
Write neatly. Simplify your answers when possible.

Part 1. 40 % of the test score.

Give the definition of

(a) (1 point) Theorem

(b) (1 point) Set of real numbers

(c) (1 point) Convergence of monotone sequence

(d) (1 point) Limit superior

(e) (1 point) Cauchy sequence

(f) (1 point) Absolutely and conditionally convergent series

Part 2. 80 % of the test score.

1. (10 points) By constructing truth table find if the proposition $\sim P \Rightarrow (P \vee \sim Q)$ is a rule of inference or not.

2. (10 points) Show that $|[0,1]| = |\mathbb{R}|$.

3. (10 points) Prove that 8 divides the number $3^{2n} - 1$ for any natural n.

4. (10 points) Show that $\inf\left\{\frac{1}{n} : n \in \mathbb{N}\right\} = 0$.

5. (10 points) Show that a convergent sequence has a unique limit.

6. (10 points) Using the definition of Cauchy sequence prove or disprove that the sequence $\left\{\frac{n^2-2n}{n^2}\right\}$ is Cauchy.

7. (10 points) Find if the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^3 - n + 1}{(-2)^n}$ is conditionally convergent, absolutely convergent, or divergent. Support your answer.

8. (10 points) Find if the series $\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{n}$ is conditionally convergent, absolutely convergent, or divergent. Support your answer.