Math 0413 **Midterm Exam**

Summer 2016 **Name:** ____________________

No calculators, no books. Show all your work (no work = no credit).
Write neatly. Simplify your answers when possible.

Part 1. 30 % of the test score.

Give the definition of

(a) (1 point) A tautology

(b) (1 point) A function

(c) (1 point) An ordered field F (assuming that F is a field)

(d) (1 point) The Archimedean property of real numbers (part 1)

Part 2. 70 % of the test score.

1. (5 points) Show that the proposition $\sim(Q \Rightarrow (P \vee \sim Q))$ is a fallacy.

2. (5 points) Negate the statement $\forall x \in \mathbb{R}\ \exists a, b \in \mathbb{R}$ such that if $x < a$ then $x < b$.

(Note: neither the statement nor its negation has to be true).

3. (5 points) For $(a, b), (c, d) \in \mathbb{Z} \times \mathbb{Z}$ define $(a, b) \sim (c, d)$ to mean that $a + d = b + c$.

Is this an equivalence relation? Support your answer.

4. (5 points) Let A be a set and $\mathscr{P}(A)$ be its power set. Show that a function $f: A \to \mathscr{P}(A)$ is not a surjection.

5. (5 points) Let F be an ordered field and $x, y, z \in F$. Show that $x \neq 0 \Rightarrow x^2 > 0$.

In a proof you may use the following properties of F:

(1) $x > 0 \Leftrightarrow -x < 0$; (2) $x > 0,\, y < z \Rightarrow xy < xz$; and (3) $x < 0,\, y < z \Rightarrow xy > xz$.

6. (5 points) By using the Archimedean property of real numbers prove that

if $t \in \mathbb{R},\ t < 0$ then $\exists n \in \mathbb{N}$ such that $-\dfrac{1}{n} > t$.