Math 0413 **Midterm Exam**

Spring 2018 **Name:** _____

No calculators, no books. Show all your work (no work = no credit).
Write neatly. Simplify your answers when possible.

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1. (10 points) Give the definition of least upper bound property of a set

2. (10 points) Negate the statement

   "For every $a, b \in \mathbb{R}$ with $a < b$ there is an $r \in \mathbb{Q}$ with $a < r < b$".

3. (10 points) Suppose $\sim$ is an equivalence relation on a set A. Show that $\forall a, b \in A \;\; [a] \cap [b] \neq \varnothing$ implies $[a] = [b]$, where $[a]$ denotes the equivalence class of the element a.

4. (10 points) Let $A = \left\{ \frac{1}{n} : \ n \in \mathbb{N} \right\}$, $B = \{2k : \ k \in \mathbb{N}\}$. Show that $|A| = |B|$.

5. (10 points) Consider the increasing sequence of real numbers $x_1 = 1$ and $x_{n+1} = \sqrt{1 + 2x_n}$ for $n \geq 1$. Use the Principle of Mathematical Induction to show that $x_n < 4 \ \ \forall n \geq 1$.

6. (10 points) Prove that if $A = \left\{1 - \frac{1}{n},\ n \in \mathbb{N}\right\}$ then $\sup A = 1$.