Math 0413 **Final Exam**

Spring 2018 **Name:** _____

No calculators, no books. Show all your work (no work = no credit).
Write neatly. Simplify your answers when possible.

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1. (10 points) By constructing truth table find if the proposition $\sim P \Rightarrow (P \vee \sim Q)$ is a rule of inference or not.

2. (a) (3 points) Give the definition of a function.

(b) (7 points) Let $f\colon\ A \to B$ be a function and $C \subset B$. Show that

$$f^{-1}(B \setminus C) = A \setminus f^{-1}(C)$$

3. (10 points) Show that $|[0,1]| = |\mathbb{R}|$.

4. (a) (3 points) Give the definition of the principle of induction.

(b) (7 points) Prove that 8 divides the number $3^{2n} - 1$ for any natural $n$.

5. (10 points) Let $a, b, x \in \mathbb{R}$. Prove that if $x \geq a \ \forall a < b$ then $x \geq b$.

6. (10 points) Show that a convergent sequence has a unique limit.

7. (a) (3 points) Give the definition of a convergent sequence.

(b) (7 points) Is the sequence $\left\{\dfrac{n}{5n+1}\right\}$ convergent? Support your answer by using $\varepsilon$ and $M$ from the definition of a convergent sequence. If the sequence is convergent then find the limit.

8. (a) (3 points) Give the definition of a Cauchy sequence.

(b) (7 points) Using the definition of Cauchy sequence prove or disprove that the sequence $\left\{\frac{n^2-7n}{n^2}\right\}$ is Cauchy.

9. (10 points) Show that if a sequence $\{x_n\}$ is convergent then it is Cauchy.

10. (10 points) Find if the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^4 - 3n + 1}{(-2)^n}$ is conditionally convergent, absolutely convergent, or divergent. Support your answer.