Summer 2016 **Practice Problems for Final Exam** Math 0413

Appendix A

1. By constructing truth tables find if the following propositions are logically equivalent

   (a) $\sim(P \wedge Q)$ and $(\sim P) \vee (\sim Q)$

   (b) $\sim(P \vee Q)$ and $(\sim P) \wedge (\sim Q)$

   (c) $\sim(P \Rightarrow Q)$ and $(\sim P) \Rightarrow (\sim Q)$

2. By constructing truth table show that the proposition $(P \Rightarrow Q) \vee (Q \Rightarrow P)$ is a tautology.
3. By constructing truth table show that the proposition $\sim P \wedge Q \Rightarrow P$ is a fallacy.
4. By constructing truth table show that the proposition (called a contrapositive) $(P \Rightarrow Q) \Leftrightarrow (\sim Q \Rightarrow \sim P)$ is a rule of inference.
5. By constructing truth table show that the proposition (called a modus ponens) $P \wedge (P \Rightarrow Q)) \Rightarrow Q$ is a rule of inference.
6. By constructing truth table show that the proposition (called a modus tollens) $\sim Q \wedge (P \Rightarrow Q)) \Rightarrow \sim P$ is a rule of inference.

Problems from the textbook: A.4.5, A.4.7, A.4.8, A.4.9

Appendix B

1. Define a relation $\sim$ on $\mathbb{Q}$ by defining $a \sim b$ to mean $ab \geq 0$. Is this an equivalence relation? Support your answer.
2. For $(a, b), (c, d) \in \mathbb{R}^2$ define $(a, b) \sim (c, d)$ to mean that $a + b = c + d$. Is this an equivalence relation? Support your answer.
3. On the set $\{(a, b)\}$ of all pairs of natural numbers, define $(a_1, b_1) \sim (a_2, b_2)$ if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$. Is this an equivalence relation? Support your answer.
4. Define a relation $\sim$ on $\mathbb{Z}$ by defining $a \sim b$ to mean $a + b = 3n$ for some integer n. In other words, the sum $a + b$ is divisible by 3. Is this an equivalence relation? Support your answer.

Problems from the textbook: B.3.1, B.3.2, B.3.4, B.3.5

Chapter 0

1. Prove both parts of the Theorem 0.3.5.
2. Prove the Theorem 0.3.6.
3. Prove that 5 divides the number $8^n - 3^n$ for any natural n.
4. Prove Propositions 0.3.15 and 0.3.16.
5. Prove that $|\mathbb{N} \times \mathbb{N}| = |\mathbb{N}|$.
6. Show that $|(0,1)| = |\mathbb{R}|$.
7. Give an example of a countable collection of infinite sets A_1, A_2, ... whose intersection is a finite set.
8. Prove the Theorem 0.3.27.

Problems from the textbook: 0.3.6, 0.3.8, 0.3.11, 0.3.12, 0.3.13, 0.3.14, 0.3.15, 0.3.16, 0.4.2, 0.4.4.

Section 1.1

1. Prove that $\sqrt{2} \notin \mathbb{Q}$.
2. Prove Propositions 1.1.8 and 1.1.9 (Note: in the text the proof is not complete).

Problems from the textbook: 1.1.2, 1.1.3, 1.1.5.

Section 1.2

1. Prove Propositions 1.2.2, 1.2.6, 1.2.7.
2. Prove that if $x < a$ $\forall a > b$ then $x \leq b$.
3. Prove that if $x \geq a$ $\forall a < b$ then $x \geq b$.
4. Prove the Theorem 1.2.4.
5. Prove the Corollary 1.2.5.

Problems from the textbook: 1.2.1, 1.2.2, 1.2.4, 1.2.9, 1.2.10.

Section 1.3

Prove Propositions 1.3.1, 1.3.2, 1.3.3, 1.3.7.

Problems from the textbook: 1.3.1, 1.3.2, 1.3.3, 1.3.4, 1.4.1.

Section 2.1

Prove Propositions and Theorems 2.1.6, 2.1.7, 2.1.10, 2.1.17.

Problems from the textbook: 2.1.3, 2.1.5, 2.1.7, 2.1.9, 2.1.11, 2.1.13, 2.1.15.

Section 2.2

Prove Propositions and Lemmas 2.2.1, 2.2.3, 2.2.5(i), 2.2.6, 2.2.7, 2.2.10, 2.2.11, 2.2.12(i).

Problems from the textbook: 2.2.2, 2.2.4, 2.2.5, 2.2.6, 2.2.7, 2.2.8, 2.2.9.

Section 2.3

Prove Theorem 2.3.5 and Proposition 2.3.6.

Problems from the textbook: 2.3.2, 2.3.3, 2.3.5, 2.3.6.

Section 2.4

Prove Proposition 2.4.4 and Theorem 2.4.5.

Problems from the textbook: 2.4.1, 2.4.2.

Section 2.5

Prove Propositions 2.5.5, 2.5.7, 2.5.8, 2.5.13.

Problems from the textbook: 2.5.1, 2.5.2, 2.5.3, 2.5.4, 2.5.7.