Midterm Exam

Summer 2011 Math 1180

100 points total **Your name:** ________________________________

No calculators. Show all your work (no work = no credit). Write neatly.

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1. Let

$$A = \begin{bmatrix} 4 & -2 & 1 \\ 0 & 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}.$$

(a) [5 points] Compute $A^T B^T - (BA)^T$.

$$A = \begin{bmatrix} 4 & -2 & 1 \\ 0 & 2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}.$$

(b) [5 points] Compute $B^T A^T - (AB)^T$.

2. (a) [5 points] Give the definition of linear dependence (LD) of a set of vectors.

(b) [5 points] Prove that vectors $\bar{\mathsf{v}}_1$, $\bar{\mathsf{v}}_2$, ... , $\bar{\mathsf{v}}_m$ in $\mathbb{R}^n$ are LD if and only if at least one of the vectors can be expressed as a linear combination of the others.

3. [10 points] Show that $\bar{\mathsf{w}}$ is in $span(\mathcal{B})$ and find the coordinate vector $[\bar{\mathsf{w}}]_{\mathcal{B}}$ if

$$\bar{\mathsf{w}} = \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix}, \qquad \mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \right\}$$

4. (a) [10 points] Prove that if a matrix A is invertible, then its inverse is unique.

(b) [5 points] Let a be a nonzero scalar. Find the inverse of

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & a \\ 0 & 0 & 1 \end{bmatrix}$$

5. [10 points] For what value(s) of a, if any, will the system

$$\begin{aligned} ax &+ y &= 1 \\ x &+ ay &= 1 \end{aligned}$$

have no solution, a unique solution, and infinitely many solutions?

6. [10 points] Find bases for $row(A)$, $col(A)$, and $null(A)$ if

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{bmatrix}.$$

7. [10 points] Find the coordinate vector of $p(x) = 12x^2 + x - 5$ with respect to the basis $\mathcal{B} = \{1 + x, 1 - x, x^2\}$ of $\mathscr{P}_2$.

8. [10 points] Prove that if the columns of a matrix B are LD, then so are the columns of AB, where A is a matrix such that the product makes sense.

9. Assume $||\bar{u}|| = \sqrt{15}$, $||\bar{v}|| = 3$, and $\bar{u} \cdot \bar{v} = 6$.

(a) [5 points] Find $||\bar{u} + \bar{v}||$.

(b) [5 points] Find $||\bar{u} - 2\bar{v}||$.

(c) [5 points] Show, that the Triangle Inequality holds for the vectors $\bar{\mathsf{u}}$ and $\bar{\mathsf{v}}$.

bonus problem [10 points extra] Let $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \cos\theta & \sin\theta \end{bmatrix}$. Prove, by mathematical induction, that for $n \geq 1$

$$A^n = \begin{bmatrix} \cos n\theta & -\sin n\theta \\ \cos n\theta & \sin n\theta \end{bmatrix}.$$