Quiz 1

Summer 2011

Math 1180

Your name: Solutions

problem 1.2 #22

1. [5 points] Determine whether the angle between the vectors $\begin{bmatrix}-3\\1\\-1\\1\end{bmatrix}$ and $\begin{bmatrix}1\\-2\\3\\4\end{bmatrix}$ is acute, obtuse or right.

The answer depends on the sign of $\cos\theta$, where θ is the angle between the vectors $\bar{u}=\begin{bmatrix}-3\\1\\-1\\1\end{bmatrix}$ and $\bar{v}=\begin{bmatrix}1\\-2\\3\\4\end{bmatrix}$.

The sign of $\cos\theta$ depends on $\bar{u}\cdot\bar{v}$ only.

$\bar{u}\cdot\bar{v} = -3-2-3+4 = -4 < 0 \Rightarrow \cos\theta < 0$ (II quadrant) $\Rightarrow$ the angle is $\boxed{\text{obtuse}}$.

2.3 #15 (modified)

2. [5 points] Describe geometrically and algebraically the span of the vectors $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$.

The vectors are LI $\Rightarrow$ they define two dimensional space or a plane.

Geometrically the span is the plane through the origin with direction vectors $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

For any vector $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ in the plane by the definition of a span

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = a\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} + b\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -b \\ 2a+b \\ 3a \end{bmatrix}$$ for some scalars a and b

$\Rightarrow b = -x, \quad a = \frac{1}{3}z, \quad y = 2a+b = \frac{2}{3}z - x$

or $x + y - \frac{2}{3}z = 0 \iff 3x+3y-2z=0$

Algebraicaly it is the plane with general equation $3x+3y-2z=0$

2.2 #28

3. [5 points] Solve the given system of equations using either Gaussian or Gauss-Jordan elimination.

$$\begin{aligned} 2w + 3x - y + 4z &= 0 \\ 3w - x + z &= 1 \\ 3w - 4x + y - z &= 2 \end{aligned}$$

Augmented matrix:

$$\left[\begin{array}{cccc|c} 2 & 3 & -1 & 4 & 0 \\ 3 & -1 & 0 & 1 & 1 \\ 3 & -4 & 1 & -1 & 2 \end{array}\right] \xrightarrow[2R_3-3R_1]{2R_2-3R_1} \begin{bmatrix} 2 & 3 & -1 & 4 & 0 \\ 0 & -11 & 3 & -10 & 2 \\ 0 & -17 & 5 & -14 & 4 \end{bmatrix} \xrightarrow{11R_3-17R_2}$$

$$\rightarrow \begin{bmatrix} 2 & 3 & -1 & 4 & 0 \\ 0 & -11 & 3 & -10 & 2 \\ 0 & 0 & 4 & 16 & 10 \end{bmatrix}$$

$$\begin{aligned} 2w+3x-y+4z &= 0 \\ -11x+3y-10z &= 2 \\ 2y+8z &= 5 \end{aligned}$$

$y = -4z + \frac{5}{2}$

$11x = 3y - 10z - 2 = -22z + \frac{11}{2} \Rightarrow x = -2z + \frac{1}{2}$

$2w = -3x + y - 4z = -2z + 1 \Rightarrow w = -z + \frac{1}{2}$

Hence the solution is (put $z = t$)

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -t+\frac{1}{2} \\ -2t+\frac{1}{2} \\ -4t+\frac{5}{2} \\ t \end{bmatrix} = t\begin{bmatrix} -1 \\ -2 \\ -4 \\ 1 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 5 \\ 0 \end{bmatrix}$$

where t is a free variable.

1.3 #26

bonus problem [5 points extra] Find the equation of the set of all points that are equidistant from the points $P = (1, 0, 2)$ and $Q = (5, 4, 2)$.

Let $R = (x, y, z)$ be a point that is equidistant from P and Q. We need to define the set of all such points.

$$[d(R,P)]^2 = [d(R,Q)]^2 \Leftrightarrow$$

$$\Leftrightarrow (x-1)^2 + (y-0)^2 + \cancel{(z-2)^2} = (x-5)^2 + (y-4)^2 + \cancel{(z-2)^2}$$

$$\Leftrightarrow x^2 - 2x + 1 + y^2 = x^2 - 10x + 25 + y^2 - 8y + 16$$

$$\Leftrightarrow 8x + 8y - 40 = 0 \quad \Leftrightarrow x + y - 5 = 0$$

The set of all such points form a plane with the equation $x + y - 5 = 0$.