

# Rational Exponents

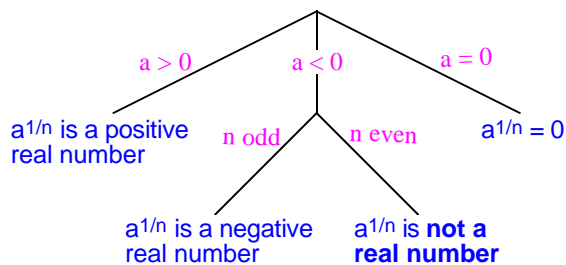
If  $a$  is a real number and  $n$  is a positive integer, we sometimes speak of the  $n^{\text{th}}$  root of  $a$ . If it exists, it should be some number  $b$  whose  $n^{\text{th}}$  power is equal to  $a$ . That is, we should have  $b^n = a$ , and then it seems reasonable to write  $b = a^{1/n}$ .

If  $a$  is a positive real number and  $n$  is a positive integer, the principal  $n^{\text{th}}$  root of  $a$  is the positive real number whose  $n^{\text{th}}$  power is equal to  $a$ . We denote the principal  $n^{\text{th}}$  root of  $a$  by using  $\frac{1}{n}$  as an exponent and we write

$$a^{1/n}.$$

To avoid any ambiguity, use the principal  $n^{\text{th}}$  root of  $a$  to calculate  $a^{1/n}$ .

If  $a$  is any real number and  $n$  is a positive integer:



**Examples:** Simplify the following expressions. Use Mathematics Writing Style!

a)  $16^{1/2}$   
= 4

b)  $125^{1/3}$   
= 5

c)  $81^{1/4}$   
= 3

d)  $\left(\frac{16}{25}\right)^{1/2}$   
=  $\frac{4}{5}$

e)  $\left(\frac{27}{8}\right)^{1/3}$   
=  $\frac{3}{2}$

f)  $\left(-\frac{1}{32}\right)^{1/5}$   
=  $-\frac{1}{2}$

g)  $8^{4/3}$   
=  $(8^{1/3})^4$   
=  $2^4$   
= 16

h)  $8^{-2/3}$   
=  $(8^{1/3})^{-2}$   
=  $2^{-2}$   
=  $\frac{1}{4}$

i)  $(-4)^{-1/2}$   
not real

j)  $25^{-3/2}$   
=  $(25^{1/2})^{-3}$   
=  $5^{-3}$   
=  $\frac{1}{125}$

k)  $-25^{-3/2}$   
=  $-(25^{1/2})^{-3}$   
=  $-5^{-3}$   
=  $-\frac{1}{125}$

l)  $3 \cdot 16^{-5/4}$   
=  $3 \cdot (16^{1/4})^{-5}$   
=  $3 \cdot 2^{-5}$   
=  $3 \cdot \frac{1}{32}$   
=  $\frac{3}{32}$

# More Rational Exponents

If  $a$  is any real number,  $m$  any nonzero integer, and  $n$  a positive integer such that  $a^{1/n}$  is defined. Then we have

$$a^{m/n} = (a^{1/n})^m = (a^m)^{1/n}$$

To calculate  $a^{m/n}$ , you should take the root first!!

$$a^{m/n} = (a^{1/n})^m.$$

# Radicals

If  $a$  is a real number, and  $n$  is a positive integer such that  $a^{1/n}$  is defined, then we have

$$\sqrt[n]{a} = a^{1/n}.$$

Thus we also have

$$a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$$

Again, to calculate  $a^{m/n}$ , you should take the root first!!

$$a^{m/n} = \left(\sqrt[n]{a}\right)^m.$$

**Examples:** Simplify the following expressions. Assume that all variables are positive.

a)  $(2x^{1/3})^3$   
=  $2^3 (x^{1/3})^3$   
=  $8x$

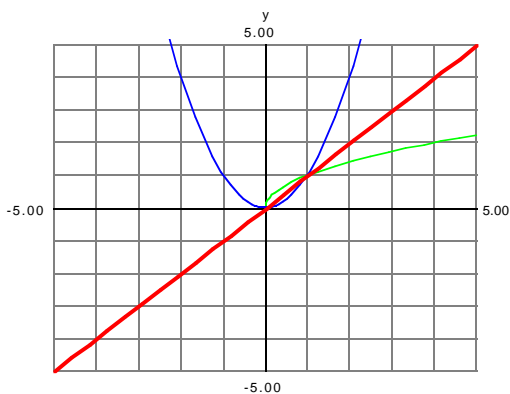
b)  $y^{2/7} \cdot y^{3/7}$   
=  $y^{(2/7 + 3/7)}$   
=  $y^{5/7}$

c)  $4z^{1/2} \cdot 3z^{1/3}$   
=  $12z^{(1/2 + 1/3)}$   
=  $12z^{5/6}$

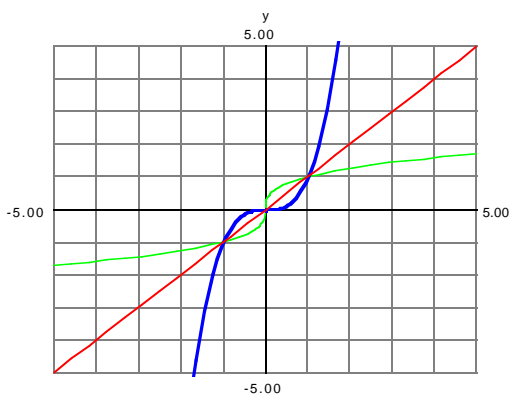
d)  $\frac{x^{2/3}}{x^{1/4}}$   
=  $x^{(2/3 - 1/4)}$   
=  $x^{5/12}$

e)  $\frac{(x^{1/4}y^{1/2})^3}{x^{1/2}y^{-1/4}}$   
=  $\frac{x^{3/4}y^{3/2}}{x^{1/2}y^{-1/4}}$   
=  $x^{(3/4 - 1/2)}y^{(3/2 + 1/4)}$   
=  $x^{1/4}y^{7/4}$

Graph of  $y = \sqrt{x}$  vs. Graph of  $y = x^2$  vs. Graph of  $y = x$



Graph of  $y = \sqrt[3]{x}$  vs. Graph of  $y = x^3$  vs. Graph of  $y = x$



## Simplifying Radicals

A radical expression is considered to be simplified when:

1. The radicand has no factors with an exponent greater than or equal to the index of the radical,

$$\sqrt[3]{x^6y^{11}} = \sqrt[3]{x^6y^9y^2} = \sqrt[3]{x^6y^9}\sqrt[3]{y^2} = x^2y^3\sqrt[3]{y^2}$$

2. No radical contains a fraction,

$$\sqrt[3]{\frac{3}{2x^4}} = \sqrt[3]{\frac{12x^2}{8x^6}} = \frac{\sqrt[3]{12x^2}}{2x^2}$$

3. No denominator contains a radical, and

$$\frac{2}{1-3\sqrt{x}} = \frac{2(1+3\sqrt{x})}{(1-3\sqrt{x})(1+3\sqrt{x})} = \frac{2(1+3\sqrt{x})}{1-9x}$$

4. The index and the power of the radicand have no common factor except 1.

$$\sqrt[9]{x^6} = \sqrt[3]{x^2}$$

## Properties of Radicals

Remember that if no index is shown, then it is by default 2.

Thus  $\sqrt[2]{x}$  means  $\sqrt{x}$ . The properties of radicals come directly from the definition and the properties of exponents. In particular, we have

### Property

### Example

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b} \quad \sqrt[3]{8x^{12}} = \sqrt[3]{8} \cdot \sqrt[3]{x^{12}} = 2x^4$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad \sqrt[4]{\frac{16}{81}} = \frac{\sqrt[4]{16}}{\sqrt[4]{81}} = \frac{2}{3}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} \quad \sqrt[5]{\sqrt{x}} = \sqrt[10]{x}$$

## Absolute Value Again

Recall that the absolute value can be defined in terms of the square root. More generally, even if we don't know the sign of  $x$ , we have for any positive integer  $n$ :

$$\sqrt[n]{x^{2n}} = |x|$$

**Examples:** Simplify the following expressions. Assume that all variables are positive. Use Mathematics Writing Style!

$$\begin{array}{llll} \text{a)} \quad \sqrt{98} & \text{b)} \quad \sqrt[3]{56} & \text{c)} \quad \sqrt[4]{243} & \text{d)} \quad \sqrt{\frac{5}{4x}} \\ = \sqrt{49 \cdot 2} & = \sqrt[3]{8 \cdot 7} & = \sqrt[4]{81 \cdot 3} & = \sqrt{\frac{5x}{4x^2}} \\ = \sqrt{49} \sqrt{2} & = \sqrt[3]{8} \sqrt[3]{7} & = \sqrt[4]{81} \sqrt[4]{3} & = \frac{\sqrt{5x}}{2x} \\ \boxed{= 7\sqrt{2}} & \boxed{= 2\sqrt[3]{7}} & \boxed{= 3\sqrt[4]{3}} & \boxed{= \frac{\sqrt{5x}}{2x}} \end{array}$$

$$\begin{array}{lll} \text{e)} \quad \sqrt[4]{\frac{16x}{9y}} & \text{f)} \quad \frac{3}{\sqrt[3]{32x}} & \text{h)} \quad \sqrt[4]{\frac{3x^5}{8y^7}} \\ = \sqrt[4]{\frac{16x \cdot 9y^3}{81y^4}} & = \frac{3}{\sqrt[3]{32x}} \cdot \frac{\sqrt[3]{2x^2}}{\sqrt[3]{2x^2}} & = \sqrt[4]{\frac{3x^5 \cdot 2y}{8y^7 \cdot 2y}} \\ \boxed{= \frac{2\sqrt[4]{9xy^3}}{3y}} & = \frac{3\sqrt[3]{2x^2}}{\sqrt[3]{64x^3}} & = \sqrt[4]{\frac{6x^5y}{16y^8}} \\ & \boxed{= \frac{3\sqrt[3]{2x^2}}{4x}} & \boxed{= \frac{\sqrt[4]{6xy}}{2y^2}} \end{array}$$

$$\begin{array}{l} \text{g)} \quad \sqrt[3]{\sqrt[4]{x}} \\ \boxed{= \sqrt[12]{x}} \end{array}$$

**Examples:** Simplify the following expressions. Assume that all variables are positive. Use Mathematics Writing Style!

$$\begin{aligned} \text{a) } 2\sqrt{10} - 6\sqrt{10} &= -4\sqrt{10} \\ \text{b) } \sqrt{32x} + \sqrt{18x} &= \sqrt{16 \cdot 2x} + \sqrt{9 \cdot 2x} \\ &= 4\sqrt{2x} + 3\sqrt{2x} \\ &= 7\sqrt{2x} \\ \text{c) } \sqrt{12} + \sqrt{18} + \sqrt{27} &= \sqrt{4 \cdot 3} + \sqrt{9 \cdot 2} + \sqrt{9 \cdot 3} \\ &= 2\sqrt{3} + 3\sqrt{2} + 3\sqrt{3} \\ &= 5\sqrt{3} + 3\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{d) } \sqrt{5} + \sqrt{45} - \sqrt{15} &= \sqrt{5} + \sqrt{9 \cdot 5} - \sqrt{15} \\ &= \sqrt{5} + 3\sqrt{5} - \sqrt{15} \\ &= 4\sqrt{5} - \sqrt{15} \\ \text{e) } \sqrt{8x} - 3\sqrt{2x} + \sqrt{18x} &= \sqrt{4 \cdot 2x} - 3\sqrt{2x} + \sqrt{9 \cdot 2x} \\ &= 2\sqrt{2x} - 3\sqrt{2x} + 3\sqrt{2x} \\ &= 2\sqrt{2x} \end{aligned}$$

$$\begin{aligned} \text{f) } \sqrt[3]{5x} - \sqrt[3]{40x} &= \sqrt[3]{5x} - \sqrt[3]{8 \cdot 5x} \\ &= \sqrt[3]{5x} - 2\sqrt[3]{5x} \\ &= -\sqrt[3]{5x} \\ \text{g) } \sqrt[4]{x^7} + x\sqrt[4]{81x^3} &= \sqrt[4]{x^4 \cdot x^3} + x\sqrt[4]{81 \cdot x^3} \\ &= x\sqrt[4]{x^3} + 3x\sqrt[4]{x^3} \\ &= 4x\sqrt[4]{x^3} \\ \text{h) } \sqrt[5]{x^5y^{10}z} + 4xy^2\sqrt[5]{z} &= xy^2\sqrt[5]{z} + 4xy^2\sqrt[5]{z} \\ &= 5xy^2\sqrt[5]{z} \end{aligned}$$

$$\begin{aligned} \text{o) } \frac{2}{4 - 3\sqrt{2}} &= \frac{2(4 + 3\sqrt{2})}{(4 - 3\sqrt{2})(4 + 3\sqrt{2})} \\ &= \frac{2(4 + 3\sqrt{2})}{16 - 9 \cdot 2} \\ &= \frac{2(4 + 3\sqrt{2})}{16 - 18} \\ &= \frac{2(4 + 3\sqrt{2})}{-2} \\ &= -(4 + 3\sqrt{2}) \\ \text{p) } \frac{31}{6 + \sqrt{5}} &= \frac{31(6 - \sqrt{5})}{(6 + \sqrt{5})(6 - \sqrt{5})} \\ &= \frac{31(6 - \sqrt{5})}{36 - 5} \\ &= \frac{31(6 - \sqrt{5})}{31} \\ &= 6 - \sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{q) } \frac{1}{2\sqrt{7} - 3\sqrt{2}} &= \frac{1(2\sqrt{7} + 3\sqrt{2})}{(2\sqrt{7} - 3\sqrt{2})(2\sqrt{7} + 3\sqrt{2})} \\ &= \frac{2\sqrt{7} + 3\sqrt{2}}{4 \cdot 7 - 9 \cdot 2} \\ &= \frac{2\sqrt{7} + 3\sqrt{2}}{28 - 18} \\ &= \frac{2\sqrt{7} + 3\sqrt{2}}{10} \\ \text{r) } \frac{2 + \sqrt{5}}{3 - \sqrt{5}} &= \frac{(2 + \sqrt{5})(3 + \sqrt{5})}{(3 - \sqrt{5})(3 + \sqrt{5})} \\ &= \frac{6 + 5\sqrt{5} + 5}{9 - 5} \\ &= \frac{11 + 5\sqrt{5}}{4} \end{aligned}$$

$$\begin{aligned} \text{i) } (\sqrt{7} - 2)(\sqrt{7} + 3) &= (\sqrt{7})^2 + 3\sqrt{7} - 2\sqrt{7} - 6 \\ &= 7 + \sqrt{7} - 6 \\ &= 1 + \sqrt{7} \\ \text{j) } (\sqrt{6} - 2\sqrt{5})(\sqrt{6} + \sqrt{5}) &= 6 + \sqrt{30} - 2\sqrt{30} - 2 \cdot 5 \\ &= 6 - \sqrt{30} - 10 \\ &= -4 - \sqrt{30} \end{aligned}$$

$$\begin{aligned} \text{k) } (\sqrt{2} + 5)(\sqrt{2} - 5) &= (\sqrt{2})^2 - (5)^2 \\ &= 2 - 25 \\ &= -23 \\ \text{l) } (\sqrt{3} + \sqrt{2})^2 &= 3 + 2\sqrt{3}\sqrt{2} + 2 \\ &= 3 + 2\sqrt{6} + 2 \\ &= 5 + 2\sqrt{6} \end{aligned}$$

$$\begin{aligned} \text{m) } (3 + \sqrt{2})^2 &= 3^2 + 2 \cdot 3\sqrt{2} + 2 \\ &= 9 + 6\sqrt{2} + 2 \\ &= 11 + 6\sqrt{2} \\ \text{n) } (\sqrt{6} + \sqrt{2})^2 &= (\sqrt{6})^2 + 2\sqrt{6}\sqrt{2} + (\sqrt{2})^2 \\ &= 6 + 2\sqrt{12} + 2 \\ &= 6 + 2 \cdot 2\sqrt{3} + 2 \\ &= 8 + 4\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{s) } \frac{4\sqrt{2}}{2\sqrt{2} + 3} &= \frac{4\sqrt{2}(2\sqrt{2} - 3)}{(2\sqrt{2} + 3)(2\sqrt{2} - 3)} \\ &= \frac{8 \cdot 2 - 12\sqrt{2}}{4 \cdot 2 - 9} \\ &= \frac{16 - 12\sqrt{2}}{8 - 9} \\ &= \frac{16 - 12\sqrt{2}}{-1} \\ &= -16 + 12\sqrt{2} \\ \text{t) } \frac{x - 5}{\sqrt{x} - \sqrt{5}} &= \frac{(x - 5)(\sqrt{x} + \sqrt{5})}{(\sqrt{x} - \sqrt{5})(\sqrt{x} + \sqrt{5})} \\ &= \frac{(x - 5)(\sqrt{x} + \sqrt{5})}{x - 5} \\ &= \sqrt{x} + \sqrt{5} \end{aligned}$$

# Solving Radical Equations

When we solve equations with radicals, sometimes we exponentiate both sides of an equation to get a second equation. There may be solutions of the second equation that are not solutions of the first. We call them an **extraneous solutions**, and it must be thrown out. Therefore, when you solve equations with radicals, **you must check your solution in the original equation.**

## Solving Equations With One Radical

Step 1. Put the radical on one side of the equation by itself.

Step 2. Raise both sides to the power corresponding to the index of the radical.

Step 3. Solve the resulting equation.

Step 4. **Check your answers into the original equation** and discard all extraneous solutions.

Step 0. If there are two radicals (same index) do this first:

- Isolate the more complicated radical on one side of the equation.
- Raise both sides to the appropriate power.
- Simplify. You should have only one radical left.

Examples: Solve the following radical equations:

a)  $x + 3 = \sqrt{x + 5}$   $x = -1$

$$(x + 3)^2 = (\sqrt{x + 5})^2$$

$$x^2 + 6x + 9 = x + 5$$

$$x^2 + 5x + 4 = 0$$

$$(x + 4)(x + 1) = 0$$

$$x = -4, -1$$

b)  $\sqrt{y^2 - 10y - 11} = 1 + y$   $y = -1$

$$(\sqrt{y^2 - 10y - 11})^2 = (1 + y)^2$$

$$y^2 - 10y - 11 = 1 + 2y + y^2$$

$$-12y = 12$$

c)  $\sqrt{x + 4} = \sqrt{3x - 2}$   $x = 3$

$$(\sqrt{x + 4})^2 = (\sqrt{3x - 2})^2$$

$$x + 4 = 3x - 2$$

$$-2x = -6$$

d)  $\sqrt{3x + 13} + 3 = 2x$   $x = 4$

$$\sqrt{3x + 13} = 2x - 3$$

$$(\sqrt{3x + 13})^2 = (2x - 3)^2$$

$$3x + 13 = 4x^2 - 12x + 9$$

$$0 = 4x^2 - 15x - 4$$

$$0 = (4x - 1)(x - 4)$$

$$x = 1/4, 4$$

e)  $\sqrt[3]{2x + 1} + 1 = 3$

$$\sqrt[3]{2x + 1} = 2$$

$$(\sqrt[3]{2x + 1})^3 = 2^3$$

$$2x + 1 = 8$$

$$2x = 7$$

$$x = 7/2$$

f)  $\sqrt[3]{2x - 9} + 4 = 3$

$$\sqrt[3]{2x - 9} = -1$$

$$(\sqrt[3]{2x - 9})^3 = (-1)^3$$

$$2x - 9 = -1$$

$$2x = 8$$

$$x = 4$$

g)  $2\sqrt{x + 4} = x + 1$  h)  $\sqrt{x} - \sqrt{2x - 14} = 1$

$$(2\sqrt{x + 4})^2 = (x + 1)^2$$

$$4(x + 4) = x^2 + 2x + 1$$

$$4x + 16 = x^2 + 2x + 1$$

$$0 = x^2 - 2x - 15$$

$$0 = (x - 5)(x + 3)$$

$$x = 5, -3$$

$$x = 5$$

$$\sqrt{x} - 1 = \sqrt{2x - 14}$$

$$(\sqrt{x} - 1)^2 = (\sqrt{2x - 14})^2$$

$$x - 2\sqrt{x} + 1 = 2x - 14$$

$$-2\sqrt{x} = x - 15$$

$$(-2\sqrt{x})^2 = (x - 15)^2$$

$$4x = x^2 - 30x + 225$$

$$0 = x^2 - 34x + 225$$

$$0 = (x - 9)(x - 25)$$

$$x = 9, 25$$

$$x = 9$$

i)  $\sqrt{3x + 1} + 1 = \sqrt{x}$

$$\sqrt{3x + 1} = \sqrt{x} - 1$$

$$(\sqrt{3x + 1})^2 = (\sqrt{x} - 1)^2$$

$$3x + 1 = x - 2\sqrt{x} + 1$$

$$2x = -2\sqrt{x}$$

$$x = -\sqrt{x}$$

$$(x)^2 = (-\sqrt{x})^2$$

$$x^2 = x$$

$$x^2 - x = 0$$

$$x(x - 1) = 0$$

$$x = 0, 1$$

$$\text{no solution}$$

# Complex Numbers

Recall that if  $x$  is any real number, then  $x^2$  is nonnegative. On the other hand, if  $\sqrt{-1}$  is defined, then its square equals  $-1$ . Hence  $\sqrt{-1}$  is not a real number. We now expand our number system by including such numbers.

**Definition:** We define  $i = \sqrt{-1}$  to be the complex unit. It follows that  $i^2 = -1$ .

**Definition:** We define a complex number to be any number of the form

$$x = a + bi,$$

where  $i$  is the imaginary unit and  $a$  and  $b$  are real numbers. In this case we call  $a$  the real part of  $x$  and  $b$  the imaginary part of  $x$ .

If you remember that  $i^2 = -1$  and otherwise treat  $i$  like any other variable, then you can do arithmetic with complex numbers in the usual way. No expression containing  $i$  raised to an integer power is considered simplified. In some sense,  $i$  can be considered to be a radical, so we never want to leave  $i$  in the denominator of a fraction.

$$\begin{array}{lll} \text{j)} & -2i(3 - i) & \text{k)} \quad (6 + 3i)(2 - 7i) \quad \text{l)} \quad (2 + 4i)(1 + i) \\ & = -6i + 2i^2 & = 12 - 36i - 21i^2 = 2 + 6i + 4i^2 \\ & = -6i + 2(-1) & = 12 - 36i - 21(-1) = 2 + 6i + 4(-1) \\ & = -6i - 2 & = 12 - 36i + 21 = 2 + 6i - 4 \\ & \boxed{= -2 - 6i} & \boxed{= 33 - 36i} \quad \boxed{= -2 + 6i} \end{array}$$

$$\begin{array}{lll} \text{m)} & (3 - 2i)^2 & \text{n)} \quad (\sqrt{2} - i)^2 \quad \text{o)} \quad (4 + 3i)(4 - 3i) \\ & = 9 - 12i + 4i^2 & = 2 - 2i\sqrt{2} + i^2 = 16 - 9i^2 \\ & = 9 - 12i + 4(-1) & = 2 - 2i\sqrt{2} + (-1) = 16 - 9(-1) \\ & = 9 - 12i - 4 & = 2 - 2i\sqrt{2} - 1 = 16 + 9 \\ & \boxed{= 5 - 12i} & \boxed{= 1 - 2i\sqrt{2}} \quad \boxed{= 25} \end{array}$$

Perform the indicated operation and /or simplify:

$$\begin{array}{llll} \text{a)} & \sqrt{-25} & \text{b)} & \sqrt{-16} \\ & = i\sqrt{25} & & = i\sqrt{16} \\ & \boxed{= 5i} & & \boxed{= 4i} \end{array}$$

$$\begin{array}{llll} \text{c)} & \sqrt{-24} & \text{d)} & \sqrt{-45} \\ & = i\sqrt{24} & & = i\sqrt{45} \\ & \boxed{= 2i\sqrt{6}} & & \boxed{= 3i\sqrt{5}} \end{array}$$

$$\begin{array}{ll} \text{e)} & (6 - 2i) + (1 - 2i) \\ & = 6 - 2i + 1 - 2i \\ & \boxed{= 7 - 4i} \end{array}$$

$$\begin{array}{l} \text{f)} \quad (-4 - i\sqrt{2}) - (-4 + i\sqrt{2}) \\ = -4 - i\sqrt{2} + 4 - i\sqrt{2} \\ \boxed{= -2i\sqrt{2}} \end{array}$$

$$\begin{array}{ll} \text{g)} & (\sqrt{3} - 2i) + (1 + i\sqrt{3}) \\ & = \sqrt{3} - 2i + 1 + i\sqrt{3} \\ & \boxed{= (1 + \sqrt{3}) + (-2 + \sqrt{3})i} \end{array}$$

$$\begin{array}{l} \text{h)} \quad 3(-7 + i\sqrt{3}) + 2(5 - 2i) \\ = -21 + 3i\sqrt{3} + 10 - 4i \\ \boxed{= -11 + (3\sqrt{3} - 4)i} \end{array}$$

$$\begin{array}{l} \text{i)} \quad 8(4 + i) - 5(5 + 2i) \\ = 32 + 8i - 25 - 10i \\ \boxed{= 7 - 2i} \end{array}$$

$$\begin{array}{ll} \text{p)} & \frac{1 + i}{2 - 3i} \\ & = \frac{(1 + i)(2 + 3i)}{(2 - 3i)(2 + 3i)} \\ & = \frac{2 + 5i + 3i^2}{4 - 9i^2} \\ & = \frac{2 + 5i - 3}{4 + 9} \\ & = \frac{-1 + 5i}{13} \\ & \boxed{= -\frac{1}{13} + \frac{5}{13}i} \end{array}$$

$$\begin{array}{l} \text{q)} \quad \frac{4}{-1 - 5i} \\ = \frac{4(-1 + 5i)}{(-1 - 5i)(-1 + 5i)} \\ = \frac{4(-1 + 5i)}{1 - 25i^2} \\ = \frac{4(-1 + 5i)}{1 + 25} \\ = \frac{4(-1 + 5i)}{26} \\ \boxed{= -\frac{2}{13} + \frac{10}{13}i} \end{array}$$

$$\begin{array}{ll} \text{r)} & \frac{7 + i}{2 - i} \\ & = \frac{(7 + i)(2 + i)}{(2 - i)(2 + i)} \\ & = \frac{14 + 9i + i^2}{4 - i^2} \\ & = \frac{14 + 9i - 1}{4 + 1} \\ & = \frac{13 + 9i}{5} \\ & \boxed{= \frac{13}{5} + \frac{9}{5}i} \end{array}$$

$$\begin{array}{l} \text{s)} \quad \frac{6 + i}{i} \\ = \frac{(6 + i)(-i)}{i(-i)} \\ = \frac{-6i - i^2}{-i^2} \\ = \frac{-6i + 1}{1} \\ \boxed{= 1 - 6i} \end{array}$$

$$\begin{aligned}
 \text{t) } & \frac{\sqrt{2} + i}{-\sqrt{2} + i} \\
 &= \frac{(\sqrt{2} + i)(-\sqrt{2} - i)}{(-\sqrt{2} + i)(-\sqrt{2} - i)} \\
 &= \frac{-2 - 2i\sqrt{2} - i^2}{2 - i^2} \\
 &= \frac{-2 - 2i\sqrt{2} + 1}{2 + 1} \\
 &= \frac{-1 - 2i\sqrt{2}}{3} \\
 &= \boxed{-\frac{1}{3} - \frac{2\sqrt{2}}{3}i}
 \end{aligned}$$

$$\begin{aligned}
 \text{u) } & \frac{5}{3 + 4i} \\
 &= \frac{5(3 - 4i)}{(3 + 4i)(3 - 4i)} \\
 &= \frac{5(3 - 4i)}{9 - 16i^2} \\
 &= \frac{5(3 - 4i)}{9 + 16} \\
 &= \frac{5(3 - 4i)}{25} \\
 &= \frac{3 - 4i}{5} \\
 &= \boxed{\frac{3}{5} - \frac{4}{5}i}
 \end{aligned}$$