Homework 1 for Math 413
Due day: Tuesday September 11 recitations.

Problem 1. Use the truth table to prove equivalence of the statements:
$p \equiv q$ and $(p \Rightarrow q) \wedge (q \Rightarrow p)$.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 2. Use the truth table to prove equivalence of the statements:

$$(p \Rightarrow q) \quad \equiv \quad \neg(p \wedge \neg q) \quad \equiv \quad (\neg q \Rightarrow \neg p).$$

Proof. WRITE YOUR SOLUTION HERE. □

Problem 3. Use the equivalence

$$p \wedge (q \vee r) \quad \equiv \quad (p \wedge q) \vee (p \wedge r) \tag{0.1}$$

to prove

$$p \vee (q \wedge r) \quad \equiv \quad (p \vee q) \wedge (p \vee r).$$

To this end apply (0.1) to $\neg p$, $\neg q$, $\neg r$ in place of p, q, r, and negate the statement using De Morgan's Laws.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 4. Negate the statement[1]

$$\forall \varepsilon > 0 \;\; \exists \delta > 0 \;\; \forall x \in \mathbb{R} \;\; \forall y \in \mathbb{R} \;\; (|x - y| < \delta \;\Rightarrow\; |\sin x - \sin y| < \varepsilon).$$

Proof. WRITE YOUR SOLUTION HERE. □

Problem 5. Negate the statement: *For all real numbers x, y satisfying $x < y$, there is a rational number q such that $x < q < y$.* Formulate the negation as a sentence and not as a formula involving quantifiers.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 6. Use an argument by contradiction prove that $\sqrt{3}$ is irrational.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 7. In Example 1.5 we provided a direct proof. Prove the same statement using a proof by contradiction.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 8. Prove the following statement[2]

$$\forall \varepsilon > 0 \;\; \exists n_0 \in \mathbb{N} \;\; \forall n \in \mathbb{N} \quad (n \geq n_0 \;\Rightarrow\; n^{-1} \leq \varepsilon).$$

Proof. WRITE YOUR SOLUTION HERE. □

Problem 9. Find a mistake in the solution of Example 1.15and provide a correct solution.

Proof. WRITE YOUR SOLUTION HERE. □

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[1] This is a true statement known as uniform continuity of the function $\sin x$. However, you are not asked to prove the statement only to negate it.

[2] Compare with Example 1.12.