Homework 2 for Math 413
Due day: Thursday September 20 recitations.

Problem 10. Prove that $A \cup B = (A \setminus B) \cup (B \setminus A) \cup (A \cap B)$.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 11. Prove that $A \cap B = (A \cup B) \setminus (A \Delta B)$, where $A \Delta B = (A \setminus B) \cup (B \setminus A)$.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 12. Let $A_t = \{x \in \mathbb{R} : |x| \leq t^2\}$, $t \in \mathbb{R}$. Find $\bigcap_{t \in \mathbb{R}} A_t$.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 13. Let $A_n = \left[1 + \frac{1}{n}, 2 + \frac{1}{n}\right]$, $n \in \mathbb{N}$. Find $\bigcap_{n=1}^{\infty} A_n$ and $\bigcup_{n=1}^{\infty} A_n$.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 14. Provide an example of a function $f : X \to Y$ and sets $A, B \subset X$ such that $f(A \cap B) \neq f(A) \cap f(B)$.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 15. Prove that if $f : X \to Y$ is one-to-one and $A, B \subset X$, then $f(A \cap B) = f(A) \cap f(B)$.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 16. Let $f : X \to Y$ and $g : Y \to Z$. Prove that if $g \circ f : X \to Z$ is one-to-one, then f is one-to-one.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 17. Provide an example of functions $f : X \to Y$ and $g : Y \to Z$ such that $g \circ f : X \to Z$ is one-to-one, but $g : Y \to Z$ is not one-to-one.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 18. Prove that $f(x) = \frac{2-x}{x}$ is a bijection from $[1, 2]$ to $[0, 1]$. (You must also include a proof that the function f is well defined i.e., that the values of f belong to the interval $[0, 1]$.)

Proof. WRITE YOUR SOLUTION HERE. □