Differential Geometry: homework # 2
Due day: September 23

All problems are graded in the scale 0–10. You need to show all your work. Answer is not enough. I am not sure if I will not return the homework, so you might want to keep a copy.

Problem 12. Let $\alpha = \alpha(t) : I \to \mathbb{R}^3$ be any regular curve three times continuously differentiable (not necessarily parametrized by arc-length). Prove that the arc-length reparametrization of α is a Frenet curve if and only if $\alpha'(t)$ and $\alpha''(t)$ are linearly independent for all t.

Problem 13. Let $L_1, L_2 : \mathbb{R}^3 \to \mathbb{R}^3$ be linear transformations with matrices L_1 and L_2 respectively. Prove that the matrix of the linear transformation $L_2 \circ L_1$ is the product of matrices $L_2 L_1$.

Problem 14. Prove that if $L : \mathbb{R}^3 \to \mathbb{R}^3$ is an orthogonal transformation, then

(a) $|Lv| = |v|$ for all $v \in \mathbb{R}^3$,
(b) $\langle Lv, Lw \rangle = \langle v, w \rangle$ for all $v, w \in \mathbb{R}^3$.

Problem 15. Let A be a 3×3 matrix. Prove that

(a) $O(3) = \{A : AA^T = I\}$,
(b) $O(3) = \{A : A^T A = I\}$

Problem 16. Prove that $A \in O(3)$ if and only if $A^T \in O(3)$.

Problem 17. Let $L : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear map and let $\alpha : I \to \mathbb{R}^3$ be a smooth curve. Then $\beta = L \circ \alpha : I \to \mathbb{R}^3$ is also a smooth curve. Prove that $\beta'(t) = L(\alpha'(t))$ for all $t \in I$.