Differential Geometry: homework # 5
Due day: November 14

All problems are graded in the scale 0–10. You need to show all your work. Answer is not enough. I am not sure if I will not return the homework, so you might want to keep a copy.

Problem 32. Let $f : \mathbb{R} \to \mathbb{R}$ be a smooth function and let

$$G_f = \{(x, f(x)) : x \in \mathbb{R}\}$$

be the graph of f. Find a diffeomorphism : $\Phi : \mathbb{R}^2 \to \mathbb{R}^2$ such that $\Phi(G_f)$ is the x-axis.

Problem 33. Prove that $\mathbb{R}^2$ is diffeomorphic to the open unit disc.

Problem 34. Prove that there is no polynomial equation of degree ≤ 3, $F(x, y, z) = 0$ that would describe the surface of the torus. **Hint:** *a line intersects the torus in four points.*

Problem 35. Find a polynomial equation of degree 6, $F(x, y, z) = 0$ which describes the union of a torus and a sphere. **Hint:** $P = 0$, $Q = 0$. *What is* $PQ = 0$*?*

Problem 36. Solve the exercise from p.189.

Problem 37. (See p. 194) Prove that

(a)
$$\pi_N(x, y, z) = (u(x, y), v(x, y)) = \left(\frac{x}{1-z}, \frac{y}{1-z}\right),$$

(b)
$$\pi_N^{-1}(u, v) = \left(\frac{2u}{u^2+v^2+1}, \frac{2v}{u^2+v^2+1}, \frac{u^2+v^2-1}{u^2+v^2+1}\right),$$

(c) $\pi_N^{-1} : \mathbb{R}^2 \to S^2 \setminus \{N\}$ is a regular parametrization.

Problem 38. Find an immersion $f : \mathbb{R}^2 \to \mathbb{R}^3$ whose image is the torus with radii $R > r$.