

Homework 4 for Math 1530

Due day: Tuesday October 1 recitations.

Problem 35. Prove that if a sequence $\{a_n\}$ is bounded and $a_{n+1} \geq a_n - \frac{1}{2^n}$ for all n , then the sequence $\{a_n\}$ is convergent.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 36. Find the limit $\lim_{n \rightarrow \infty} n \sin(2\pi en!)$.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 37. Find the limit

$$\lim_{n \rightarrow \infty} n^3 \left(\sqrt{n^2 + \sqrt{n^4 + 1}} - n\sqrt{2} \right).$$

Proof. WRITE YOUR SOLUTION HERE. □

Problem 38. Suppose that $\{a_n\}_n$ is a sequence such that for every integer $k \geq 2$ the sequence $\{a_{k \cdot n}\}_n$ is convergent. Does it follow that the sequence $\{a_n\}_n$ is convergent?

Proof. WRITE YOUR SOLUTION HERE. □

Problem 39. Prove that if $\{a_n\}_n$ is a sequence such that each of the sequences $\{a_{2n}\}_n$, $\{a_{2n+1}\}_n$ and $\{a_{3n}\}_n$ is convergent, then $\{a_n\}_n$ is convergent too.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 40. Prove that there is a sequence such that the set of all possible limits of subsequences is the whole interval $[0, 1]$.

Proof. WRITE YOUR SOLUTION HERE. □

Problem 41. Prove that if $\lim_{n \rightarrow \infty} a_n = a \in \mathbb{R}$, then for any sequence $\{b_n\}$

$$\limsup_{n \rightarrow \infty} (a_n + b_n) = a + \limsup_{n \rightarrow \infty} b_n.$$

Proof. WRITE YOUR SOLUTION HERE. □

Problem 42. Let c_0 be the class of all sequences $\{x_n\}$ such that $\lim_{n \rightarrow \infty} x_n = 0$. Prove that if $\{a_n\}$ is a bounded sequence, then

$$\inf_{(x_n) \in c_0} \left(\sup_{n \in \mathbb{N}} \{a_n + x_n\} \right) = \limsup_{n \rightarrow \infty} a_n.$$

Proof. WRITE YOUR SOLUTION HERE. □