Analysis 1: homework # 2
Due day: Friday September 4, 2020

NAME (print):

If you do **not** have a complete solution do not submit it as you will get negative points for an incomplete solution. All solutions have to be written in LaTeX using this template and submitted as a pdf file.

Problem 8. Let $\mathcal{C}$ be the collection of all uncountable sets in $\mathbb{R}$. Prove that the σ-algebra generated by $\mathcal{C}$ is the power set $\mathcal{P}(\mathbb{R})$.

Proof. (Write your solution here.) $\square$

Problem 9. Prove that the σ-algebra generated by intervals (a, ∞), $a \in \mathbb{R}$ coincides with the σ-algebra of all Borel sets $\mathfrak{B}(\mathbb{R})$.

Proof. (Write your solution here.) $\square$

Problem 10. Prove that the intersection of two F_σ-sets is F_σ.

Proof. (Write your solution here.) $\square$

Problem 11. Prove that the set of rational numbers is not G_δ.

Proof. (Write your solution here.) $\square$

Problem 12. Prove that if $0 < s < \infty$ and $E \subset X$ is a subset of a metric space, then

$$\mathcal{H}^s(E) = 0 \quad \text{if and only if} \quad \mathcal{H}^s_\infty(E) = 0.$$

Remark. Recall that $\mathcal{H}^s_\infty(E) = \inf \frac{\omega_s}{2^s} \sum_{i=1}^\infty (\operatorname{diam} A_i)^s$, where the infimum is taken over all coverings $E \subset \bigcup_{i=1}^\infty A_i$, $\operatorname{diam} A_i < \infty$.

Proof. (Write your solution here.) $\square$

Problem 13. Show an example of a set such that $\mathcal{H}^s(E) > \mathcal{H}^s_\varepsilon(E)$ for some $s > 0$ and all $\varepsilon > 0$.

Proof. (Write your solution here.) $\square$