Calculus of Variations: homework # 1
Due day: September 8, 2014

NAME (print):

Circle the problems that you have solved:

1 2 3 4 5

The solutions must be written in a **legible** form. The front page **must** be returned. All the papers **must** be stapled. If any of the conditions will not be satisfied, the homework will be disregarded. The homework **will not** be returned to the students.

Problem 1. Let $\Omega \subset \mathbb{R}^n$ be a bounded domain. Consider the functional

$$I_p(u) = \int_\Omega |\nabla u|^p, \quad p > 3$$

over the class of $C_g^\infty(\overline{\Omega})$. Find the Euler-Lagrange equations that must be satisfied by the minimizer $\bar{u} \in C_g^\infty(\bar{\Omega})$ (assuming it exists) of I_p.

(The assumption that $p > 3$ is only for technical reasons - it might simplify your computations, so do not waste too much time what thinking is the meaning of this condition. There is none.)

Problem 2. Prove that the Dirichlet integral $I(u) = \int_\Omega |\nabla u|^2$ defines a convex functional. How about $I_p(u) = \int_\Omega |\nabla u|^p$, $p > 1$?

Problem 3. Prove that if $I : X \to \mathbb{R}$ is a sequentially weakly lower semicontinuous function defined on a Banach space, then it is lower semicontinuous.

Problem 4. Provide an example of a function $f : \mathbb{R} \to \mathbb{R}$ that is lower semicontinuous, but not continuous.

Problem 5. Provide an example of a function $f : \mathbb{R}^2 \to \mathbb{R}$ which is Gateaux differentiable at $(0, 0)$, but not Frechet differentiable.