Gravitation and the Motion of the Planets (Ch. 4)

Planetary Motion

Planets generally move eastward from night to night compared with the background stars. sometimes move west to east exhibiting retrograde motion. Retrograde motion is difficult to explain in an Earth centered Solar system. As observational accuracy increased, Earth centered models became increasingly complex.

In modern times it is Copernicus who is credited with advancing the notion that the planets (including the Earth) revolve around the sun.

Problems:

Why don't stars move? [Stars VERY distant]

Still inaccuracies in accounting for planetary motion [Planets move in elliptical orbits, not circular.

In a Sun centered solar system, RETROGRADE motion arises as a natural consequence of the different orbital periods of the Earth and the Planets (Fig. 4.3).

Synodic and Sidereal Periods.

Sidereal Period: Orbital period of planet about the Sun as defined by the distant stars.

Synodic Period: Defined as the the time elapsed between identical configurations of the Earth, Sun, and Planet.

Kepler's Laws

Using careful observations of planetary positions made by Tycho, Kepler discovered 3 laws of Planetary motion.

I. Planets move in elliptical orbits with the sun at one focus.

II. The orbital speed of the planet varies so that a line joining the Sun and the planet sweeps out equal areas in equal times

III. The orbital period (P) of a planet about the Sun is related to the orbital size (=a = semi major axis) by

$$P^2 = a^3$$

(With P in years, a in astronomical units)

Galileo's Observations

1. Moon was cratered, and had mountains
2. Sun shows sunspots.
3. Four small moons were seen to be orbiting Jupiter.
4. Venus shows phase like the moon - easily explained in a sun centered solar system.
5. His telescope revealed that there were many more stars than previously believed.

Inertia

Tendency of a body at rest to remain at rest, and

for a body in motion to remain in motion.

Newtons First Law of Motion (Law of Inertia)

A body remains at rest, or moves in a straight line at constant speed, unless acted on by an outside force.

Mass

A measure of a body's inertia --- the amount of matter an object contains. The SI unit for mass is kg.

Mass is an intrinsic property of an object. The mass of an object on the Earth is the same as its mass on the Moon. Mass can be measured by comparing the object to objects of known mass using a "balance."

Weight

The force with which an object presses onto the ground. (Thus SHOULD have the units of force -- kg m s^{-2})

Weight is *not* an intrinsic property of a body. An object weighs less on the Moon than on the Earth. A spring scale measures weight, not mass.

Mass and Weight are NOT the same

Acceleration

A measure of the change [per unit time] in velocity of an object. A change in velocity can occur because of a

(1) change in speed or a

(2) change in direction

NB To specify the motion of an object moving uniformly we need to specify **both** its speed and its direction - i.e. its velocity.

To produce an acceleration of an object a force must act on it.

Newtons Second Law of Motion

When a force **F** acts on a body of mass **m** it will experience an acceleration **a** given by

$$\mathbf{F = m\,a}$$

Newtons Third Law of Motion

When one body exerts a force on a second body, the second body exerts **an equal and opposite** force on the first body.

eg Two Skateboarders.

Gravitational force exerted by Earth on Moon, is exactly the same as the gravitational force exerted by the Moon on the Earth**.**

Newtons Law of Gravity

Two bodies attract each other with a force that is directly proportional to the product of their masses, and inversely proportional to the square of the distance, (r) between them.

$$F = \frac{G M m}{r^2}$$

G is the gravitational constant NB. While the force acting on the two bodies is identical, the acceleration of the smaller body is largest.

$$F = ma = G M m / r^2$$

Thus

$$a = G M / r^2$$

Surface Gravity (g).

A measure of the acceleration of an object at the planet (or stars) surface.

From Newtons Law of Gravity and the Second law of motion, the acceleration is independent of the mass of the object, hence given the special symbol **g**.

Let M= Mass of planet, R= Radius of planet. Then

$$g = GM / R^2$$

$g=9.8\ m/s^2$ for Earth.

g for the Moon is a factor of 6 smaller than g for the Earth.

Conservation of Energy

The total energy of an enclosed system is constant. Energy may be converted from one form to another but not created or destroyed.

Energy --- the ability to do work.

Kinetic energy --- energy of motion = $E_k = 1/2\ m\ v^2$

Potential Energy --- $E_p=-GMm/r$

Conservation of Momentum

Total momentum of particles in a system remains unchanged.

Linear momentum

=mv (vector)

Thus the linear momentum of an object is constant unless it is acted on by an EXTERNAL force.

Angular momentum (L).

The angular momentum (L) of an object moving with velocity V with respect to an axis at distance r is given by

$$L = m\ r \times v$$ (cross product)

Kepler's 2nd law is simply an expression of conservation of Angular momentum.

Orbital Motion

Whenever we throw a ball (or any object) horizontally, the ball's path describes an arc as the ball is pulled to the Earth.

As we throw the ball faster the distance travelled by the object increases.

If we throw a ball fast enough (but not too fast) the ball will orbit the Earth because the arc will match the shape of the Earth. The ball is then said to be in **Orbit**.

The centripetal acceleration (a_r) required to keep the ball in orbit with velocity v is

$$a_r = v^2/r$$

Using F=ma implies that the orbital velocity is given by

$$v = (GM / r)^{1/2}$$

where

M = mass of the body the object is in orbit about

r = distance of object from center of body.

NB. The orbital velocity is *independent* of the objects mass.

Escape velocity

A measure of the velocity required by an object to escape the gravitational pull of the planet (or star). The escape velocity, V_{esc}, is defined such that it is the *minimum* speed an object needs so that it will never fall back to the planets surface.

At infinity $E_k+E_p=0$, since object just makes it to infinity.

As object is fired from the Earth

$$E_k = 1/2\ m\ v^2$$

$$E_p=-GMm/r$$

Since energy is conserved we have $E_k+E_p=0$ hence

$$1/2\ m\ v^2 = GMm/r$$

Thus (for a spherical Earth)

$$\mathbf{V_{esc}= (2GM / R)^{1/2}}$$

where M is the Mass of the planet, and R its radius.

Generalization of Kepler's Third Law.

Two masses orbit about a common center of mass such that

$$\mathbf{P^2=4\pi^2a^3/G(M_1 + M_2)}$$

Center of Mass

$$\mathbf{M_1r_1 = M_2r_2}$$

For a circular orbit, **a** is the separation between the two masses.