Time-Series-Cross-Section Data Analysis

Spatio-Temporal Models II

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Outline

Stationarity Review

For a first-order spatial lag model,

$$\mathbf{y} = \rho \mathbf{W}\mathbf{y} + \mathbf{X}\beta + \varepsilon,$$

stationarity requires $1/\omega_{\text{min}} < \rho < 1/\omega_{\text{max}}$.

For the first-order temporal lag model,

$$y_t = \phi y_{t-1} + \boldsymbol{\beta}\mathbf{x} + \varepsilon$$

stationarity requires $1 < \phi < 1$.

Stationarity Conditions for Spatiotemporal Models

For a first-order spatio-temporal lag model,

$$\mathbf{y} = \phi \mathbf{L}\mathbf{y} + \rho \mathbf{W}\mathbf{y} + \mathbf{X}\beta + \varepsilon,$$

stationarity requires

$$|\phi| < 1 - \rho\omega_{\text{max}} \text{if} \rho \geq 0$$
$$|\phi| < 1 - \rho\omega_{\text{min}} \text{if} \rho < 0$$

If the spatial and temporal dependence are positive, the requirement is simply $\phi + \rho < 1$.

Do FEs account for Spatiotemporal Dependence?

- It is common to see researchers use unit and (or) period fixed-effects to 'account for' spatial or temporal dependence.
- Unit indicators absorb long-run, time-invariant spatial clustering in outcomes, plus any other time-invariant unobserved unit-specific factors.
- Period indicators account for 'global' shocks: spatially-invariant, uniform common across all units, additive mean-shifts.

Fixed effects are rarely (if ever) good substitutes for spatial and temporal lags; they can be used together, but one has to be careful in small samples.

The Nickell Bias

Figure: Nickell Bias in Small Samples

Table 1
OLS and LSDV bias estimates[a]

T	γ	γ bias		β	β bias	
		OLS (S.E.)	LSDV (S.E.)		OLS (S.E.)	LSDV (S.E.)
5	0.2	0.225 (0.039)	−0.147 (0.040)	0.8	−0.098 (0.044)	0.006 (0.045)
	0.8	0.049 (0.026)	−0.504 (0.058)	0.2	−0.005 (0.055)	−0.027 (0.070)
10	0.2	0.225 (0.032)	−0.059 (0.023)	0.8	−0.099 (0.031)	0.015 (0.026)
	0.8	0.049 (0.017)	−0.232 (0.032)	0.2	−0.007 (0.037)	0.002 (0.045)
20	0.2	0.225 (0.028)	−0.027 (0.015)	0.8	−0.100 (0.023)	0.009 (0.017)
	0.8	0.049 (0.012)	−0.104 (0.019)	0.2	−0.008 (0.026)	0.006 (0.028)
30	0.2	0.226 (0.026)	−0.017 (0.012)	0.8	−0.100 (0.019)	0.006 (0.014)
	0.8	0.049 (0.011)	−0.066 (0.014)	0.2	−0.008 (0.020)	0.006 (0.022)

[a] 1000 draws; $N = 100$; $\sigma_{\epsilon} = 1$; $\sigma_{\xi} = 2$; $\rho = 0.5$.

The Nickell Bias

Figure: Best Practice

Summary of recommendations			
	$T \leq 10$	$T = 20$	$T = 30$
Balanced panel	LSDVC	LSDVC	LSDVC
Unbalanced panel	GMM1	GMM1 or AH	LSDV

Smith's Bias

Figure: Smith Bias in Small Samples (SAR)

Table 2 Mean Values of Parameter Estimates for the Spatial Lag Model

Average link density	Mean $\hat{\rho}(\rho = 0.5)$	Mean $\hat{\beta}_0(\beta_0 = 1)$	Mean $\hat{\beta}_1(\beta_1 = 2)$	Mean $\hat{\beta}_2(\beta_2 = 3)$	Mean $\hat{\sigma}^2(\sigma^2 = 1)$
0.30	0.481	1.138	1.978	3.003	0.92887
0.50	0.454	1.336	1.946	2.9973	0.92644
0.80	0.369	1.942	1.922	3.0143	0.91802
0.90	0.168	3.384	1.944	2.9201	0.91908
0.95	0.033	4.302	1.985	2.9209	0.90547
0.99	– 0.830	10.330	1.994	3.012	0.89564
1.00	– 48.999	351.020	0.00004	0.00006	3.8e–010

Smith's Bias

Figure: Smith Bias in Small Samples (SEM)

Table 3 Mean Values of Parameter Estimates for the Spatial Error Model

Average link density	Mean $\hat{\rho}(\rho = 0.5)$	Mean $\hat{\beta}_0(\beta_0 = 1)$	Mean $\hat{\beta}_1(\beta_1 = 2)$	Mean $\hat{\beta}_2(\beta_2 = 3)$	Mean $\hat{\sigma}^2(\sigma^2 = 1)$
0.30	0.195	1.064	2.010	2.939	0.937
0.50	– 0.038	1.040	1.960	2.958	0.933
0.80	– 0.801	0.956	2.039	2.039	0.904
0.90	– 1.880	0.997	2.011	3.006	0.864
0.95	– 2.281	0.998	1.994	3.037	0.823
0.99	– 6.363	1.047	1.945	3.032	0.706
1.00	– 48.999	1.025	1.985	2.999	0.159