STADL Up! The Spatio-Temporal Autoregressive Distributed Lag Model for TSCS Data Analysis*

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Abstract

Time-series cross-section (TSCS) data are prevalent in political science, yet many distinct challenges presented by TSCS data remain under-addressed. We focus on how dependence in *both* space and time complicates estimating *either* spatial or temporal dependence, dynamics, and effects. Current understandings of the problems induced by neglecting temporal or cross-sectional dependence derive from considerations of one-way time-serial or cross-sectional data or stylized two-way TSCS data, with dependence assumed in only one dimension. Little is known about how modeling (well) one of temporal or cross-sectional dependence while (relatively) neglecting the other affects results in TSCS analysis. We demonstrate how such (relative) omission or misspecification will inflate estimates of the included (or better-specified) dependence parameters, attenuate other dependence-parameter estimates, and bias estimates of the effects of other model covariates. We recommend a spatiotemporal autoregressive distributed lag (STADL) model with distributed lags in *both* space and time as a reasonably general and broadly effective starting point for TSCS model-specification. We also provide code to facilitate researchers' implementation of STADL specifications for their TSCS data analyses, including routines to automate the creation of standard spatial-lag weighting matrices ($\mathbf{W}$s), estimate STADL models, and calculate appropriate spatiotemporal effects.

Key Words: time-series cross-section (TSCS) or panel data, spatial dependence, temporal dependence, spatiotemporal dependence, autoregressive distributed lag models, spatial lag, lagged dependent variable, model selection

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1 Introduction

Stimson (1985) introduced political science to the promise and peril of 'regression in space and time,' heralding a boom in research utilizing space-time data. In the 35 years since, panel and time-series cross-section (TSCS) data have come to dominate quantitative empirical analyses in political science. Figure 1 illustrates with the yearly count of keyword text-identified TSCS articles appearing in the *American Political Science Review (APSR)*, *American Journal of Political Science (AJPS)*, and *Journal of Politics (JOP)* from 1980 to 2019.[1] In recent years, 2012-2019, at least 201 articles, nearly 1 of every 8, 25 per year, and 33 in 2019 alone, contained TSCS data analysis.[2] Indeed, TSCS data-analyses have grown by now to dominate empirical political science.[3,4] Yet, few of these TSCS articles, these analyses of data in 'space *and* time', seem to meaningfully consider *both* temporal and spatial dependence. Of the 33 TSCS articles in 2019, only 12 used keywords indicative of considering temporal dependence, and only 2 of considering spatial dependence, meaning at most 2 could have jointly considered both temporal and spatial dependence, as we will argue and demonstrate is crucial. Indeed, manual review of 201 TSCS articles from 2012 to 2019 confirmed that only about 94 modeling temporal dependence directly,[5] only about 23 modeling spatial dependence directly[6], and merely 12, less than 6%, modeling both temporal and spatial dependence directly, as we will ultimately recommend.

[1]Of the 7336 articles in *APSR*, *AJPS*, and *JOP* from 1980 to 2019, we counted those containing keyword roots *time series cross section*, *panel data*, and *TSCS*. JSTOR's API data covers from 1980 to 2014 for APSR, 2015 for JOP, and 2018 for AJPS. For more-recent years, we scraped the text directly from each journal's website. Our keyword roots for 'time-series articles' were [*temporal* or *serial* or *time*] [*series* or *serial* or *autocorrelation* or *correlation* or *dependence* or *dynamics* or *lag(ged)* or *lagged dependent*], and for 'spatial analysis': *spatial* [*dependence* or *interdependence* or *autocorrelation* or *correlation* or *correlated* or *lag(ged)*], *spatial-lag dependent*, or *spatially lagged dependent*. More details available in the Appendix.

[2]Specifically: of the 1745 total articles 2012-2019, keywords identified 277 using TSCS (almost 16%, or about 1 in 6); manual skims confirmed 201 of 277 (76%, or 11.5%=almost 1 in 8 of total) having TSCS data-analysis.

[3]Also indicative of this predominance: Beck & Katz (1995)'s "What to do (and not to do) with time-series cross-section data" is the most cited article ever published in the *APSR* (as per *CrossRef* and *Web of Science*).

[4]Strictly speaking, *all* data are TSCS, given that *anything* is observed in a place at a time, so TSCS refers to the dataset's dimensionality being of greater than 1 time period *and* 1 (spatial) unit.

[5]By *directly* we mean via inclusion of time-lags; of the rest, 75 use only some time-indicator, time-trend, &/or differencing strategy, 16 used some other strategy (e.g., time-period random-effects), 4 combine Newey-West standard errors with these other strategies, and 16 seemed to employ no address of temporal correlation at all.

[6]I.e., 23 used spatial lags; of the rest, 118 use only some unit fixed-effect strategy, 6 use some spatial random-effects, 23 apply clustered or panel-corrected standard-error adjustment, leaving 31 with no apparent address of spatial association.

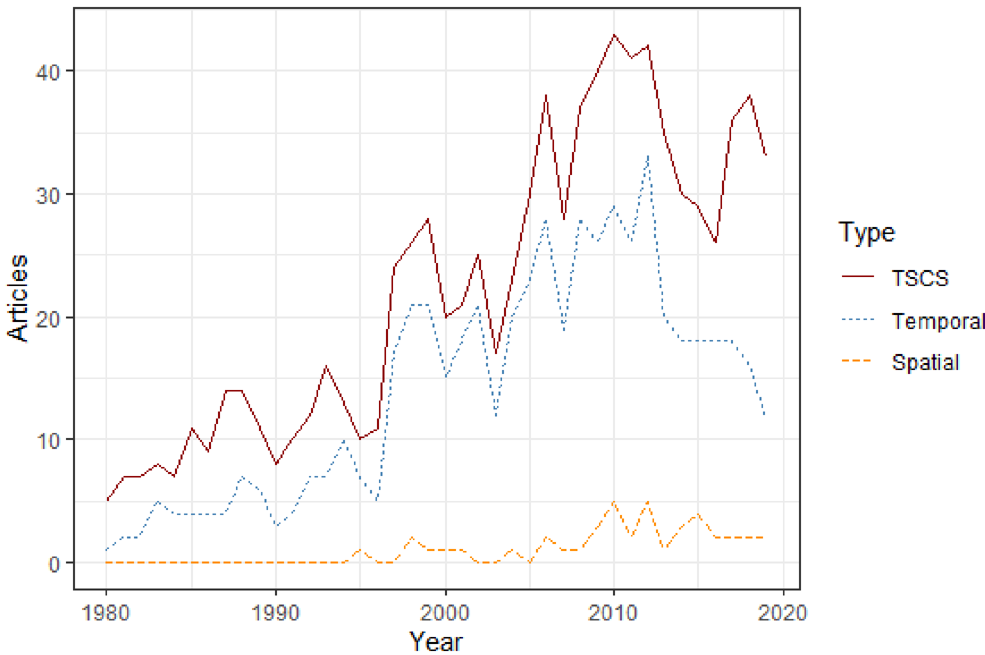


Figure 1: Count of articles using TSCS data in the 'Top-3' general PS journals, 1980-2019

Methodologically too, notwithstanding this prevalence of TSCS data in applied empirical political science, many of the unique statistical challenges of TSCS data-analysis remain un- or under-addressed. In particular, insufficient attention has been paid to the two-dimensional dependence that manifests in spatiotemporal data. Instead, just as applied research typically directly addresses dependence only in one dimension, time or, less commonly, space, borrowing strategies from time-serial or spatial-statistical methods designed for unidimensional data, the methodological literature also has generally given very little consideration to two-dimensional *Spatio-Temporal* dependence and its implications for diagnostics, specification, estimation, and inference.[7] Our understandings of temporal and spatial dependence derive almost exclusively from evaluations of one-way models that address time-serial *or* cross-sectional dependence in data that assume away the other dimension of dependence or that assume one dimension of dependence can be adequately addressed orthogonally to the emphasized other dimension.

Both applied researchers and political methodologists have generally operated as though strate-

[7] Some do briefly mention issues of two-dimensional dependence (e.g., Beck & Katz (1995), Beck & Katz (1995), Wilson & Butler (2007), Franzese & Hays (2007), Beck & Katz (2011)); however, none explore the issues we discuss here. This paper focuses on proper simultaneous specification of temporal *and* spatial dependence, i.e., on the *dimension*, space and/or time, of (inter)dependence in TSCS data. Cook et al. (2020) focuses instead on proper specification of the *source* of (primarily) spatial interdependence, i.e., in y, $\mathbf{X}$, and/or ε.

gies of addressing dependence in time-series or spatial analysis extend directly to TSCS data-analysis without requiring significant further consideration. However, when both types of dependence are appreciably present, as would always be expected in real-world TSCS data, a more complex set of relationships manifests because temporal and spatial dependence are *necessarily* related, and, therefore, cannot generally be safely considered separately. Omission or inadequate address of spatial or temporal dependence will bias estimates of dependence parameters, covariate coefficients, and dynamic & total effects. Furthermore, mismodeled spatiotemporal dependence also compromises standard diagnostic tests used to guide model specification. To be specific, omission or inadequate address of *either* spatial *or* temporal dependence leads to biases in the estimated coefficients on *both* temporal *and* spatial lags, induces biases in the coefficients on other covariates, $\mathbf{X}$, and thereby biased estimates of spatiotemporal *effects*, both of the spatiotemporal dynamic responses of outcomes, $\mathbf{y}$, over time and across spatial units, and of the instantaneous and cumulative outcome responses to any hypothetical/counterfactual.

Most importantly, because of these intertwined biases from inadequate address of *either* spatial or temporal dependence, crucial political-science substance is at stake in modeling well *both* the temporal and spatial processes inherent in TSCS data. Consider, for instance, the well-known 'development & democracy' (Lipset 1959) and 'democratic dominoes' (Starr 1991) propositions. We know that more-developed political-economies are more likely to become and to be democracies, and far more likely to *remain* democracies: temporal dependence (Przeworski et al. 2000; Robinson 2006). We also know that democracy clusters spatially, specifically geographically: "Since 1815, the probability that a randomly chosen nation would be a democracy is about 0.75 if a majority of its neighbors are democracies, but only 0.14 if a majority of its neighbors are non-democracies" (Gleditsch & Ward 2006): spatial dependence. In identifying (testing) or estimating spatial and temporal dependence, however, we immediately confront two broad challenges. First is the *source* of the dependence: i.e., spatiotemporal dependence may arise in the outcome, $\mathbf{y}$ (an autoregressive (in y) process), and/or in the observed covariates (exogenous explanators), $\mathbf{x}$ (a distributed-lag process), and/or in the unobserved/unmodeled residual, ε (an error-dependence process). In this substantive example, regarding temporal dependence, democracies may persist because accumulating experience with democracy reinforces its institutionalization (autoregressive in y), because economic development causes democracy contemporaneously and economic development persists ($x_t \longrightarrow y_t$, with x serially correlated) or because a past history of development contributes to a democratic present ($x_{t-s} \longrightarrow y_t$, a distributed lag in x), and/or because some unobserved/unmodeled covariate of democracy, culture perhaps, persists or has persistent effect on democracy (serial dependence in ε, autoregressive or distributed lag (moving average)). Similarly, the observed spatial association or clustering of democracy may arise simply because economic development causes democracy ($x_i \longrightarrow y_i$) and development clusters spatially: *clustering in observed covariates*; because developed or underdeveloped neighbors spur/stabilize or impede/destabilize democracy at home (*spatial-lag* $\mathbf{x}$, $\mathbf{x_{j \neq i}} \longrightarrow \mathbf{y_i}$, a spatial distributed-lag process): *spillovers* or *externalities* from observed covariates; because of clustering through unobserved/unmodeled external or foreign factors: *clustered unobservables* or *spatially correlated errors*;[8] and/or because foreign democracy directly influences domestic democracy ($\mathbf{y}_{j \neq i} \longleftrightarrow y_i$, a spatially autoregressive process): *contagion* or *interdependence*. In this last case, democracy itself is contagious; democracies in some units *cause* democracy in others, perhaps by demonstration effects (aspirational for pro-democracy forces and/or of costs to democracy-resisting forces, for instance) or by direct influences, e.g. of policy, from (non)democracies j on (non)democracy i.

Secondly, as we elaborate and emphasize below, distinguishing and estimating well *both* dimensions of dependence, spatial and temporal, is likewise essential to obtaining creditable tests and good estimates of the causal, spatiotemporally dynamic, and cumulative (steady-state) effects of substantive-theoretical interest.[9] Valid tests of whether and good estimates of how development affects democracy, to continue our example, will require proper specification of *both* spatial and temporal dependence processes.[10] This is because, as we have shown elsewhere (Franzese &

[8]Conceptually, spatial dependence in *unobservables* or *errors* may arise from clustering, spillovers, &/or contagion, but the specific modality of *unobserved* dynamics in *unobserved* factors is not easily discerned empirically.

[9]The order of the dependencies, i.e., the number of temporal or spatial lags, is also important but not a focus here. Time periodization of most TSCS data-analyses in political science is annual, and at that coarse temporal granularity first-order time-lags appear to suffice in most applications. Multiple spatial-lag models bring greater complications, which we do not discuss here (see Hays et al. 2010).

[10]For applied purposes, *proper* here means sufficiently well as to render remaining unaccounted spatiotemporal dependence unimportant among the inevitable modeling imperfections.

Hays 2007, 2008*a*,*b*; Cook et al. 2020), these different forms of spatiotemporal dependence imply substantively importantly different *effects*, meaning how outcomes, $\mathbf{y}$, respond to hypothetical or counterfactual 'shocks', $d\mathbf{x}$, i.e. we define *effect* $\equiv \frac{d\mathbf{y}}{d\mathbf{x}}$. Temporal or spatial dependence in outcomes $\mathbf{y}$ are autoregressive processes, which imply geometrically (exponentially) fading or accumulating dynamics and (long-run) steady-state multipliers: a democratization event in one country at some time propagates forward in time infinitely, fading geometrically, and reverberates around through neighboring countries, and then neighbors of neighbors (including bouncing back to the original country: you are your neighbors' neighbor), and neighbors of neighbors' neighbors (which include the original neighbors), and so on infinitely (again, fading geometrically). With spatial spillovers or temporal 'spill-forwards' in x, in contrast, i.e. with spatiotemporal distributed-lag processes, some increase in economic development in one country-time, spills democratizing influence forward in time however many periods there are time lags and disappears beyond that, and spills over into whatever neighboring countries, and ends there, *without* the autoregressive reverberation further forward in time and outward & back into neighbors' neighbors and so on. Autoregressive processes involve geometrically propagating dynamics and long-run temporal &/or steady-state spatial multiplier effects; distributed-lag processes have merely discretely decaying dynamics and effects, with no multipliers; and error-dependence processes, for their part, are *orthogonal*, i.e. *unrelated*, to x and so to *effects*. With spatiotemporal dependence in errors, effects of $\mathbf{x}$ on $\mathbf{y}$ are spatiotemporally static, and equal simply the coefficient on x.

Our suggested Spatio-Temporal Autoregressive Distributed Lag (STADL) model, which follows on and builds from Elhorst (2001, 2014), spans these dependence source and dimension possibilities—i.e., the STADL nests within it most common spatial, temporal, and spatiotemporal specifications—enabling proper address of *both* spatial and temporal dependence and therefore valid tests and good estimates of spatiotemporal dynamic effects, making the STADL an effective starting point for researchers' TSCS data-analyses.

2 Spatial, Temporal, and Spatiotemporal Dependence

The issues of spatial and temporal dependence *separately* have received considerable attention elsewhere, including by political scientists (e.g., Box-Steffensmeier et al. 2014, Franzese & Hays 2007, 2008*a*), so readers likely have some familiarity with both the statistical importance and the practical challenges of accounting for dependence in political-science TSCS data. These previous considerations, however, have generally confined attention to a single dimension of dependence, time *or* space, by considering only time-serial or cross-sectional contexts or, in TSCS contexts, by assuming independence on the non-focal dimension or that its dependence adequately addressed otherwise, so as to focus exclusively on temporal or spatial dependence (e.g., Beck & Katz 1995, Franzese & Hays 2007). With TSCS data, though, researchers not only inherit the challenges of both spatial (cross-unit) and temporal (over-time) dependence but also uniquely confront spatiotemporal (cross-unit, over-time) dependence as well. This section briefly reviews the conventional separate understandings of spatial and temporal dependence, focusing primarily on source (as opposed to *order*: see note 9) considerations. We then demonstrate that, in TSCS data, spatial and temporal (and spatiotemporal) dependence are *necessarily* intertwined and therefore should be considered jointly simultaneously, before offering in the next section the STADL as a practical & effective strategy for doing so.

2.1 Spatial Dependence

Cross-sectional or spatial dependence—meaning *nearby* units have more (or less) similar realizations than expected by chance alone—will be present whenever multiple units are observed in a non-random sample.[11] If *near* is defined geographically, for instance, mappings of variables with positive spatial dependence invariably exhibit geospatial clustering of so-called *hotspots* or *coldspots*. Such spatial dependence can arise because units share common traits or exposure (i.e., clustering in observed covariates or exogenous spillovers: development clusters or foreign

[11]Indeed, even in random samples, e.g. scientific surveys or randomized samples of experimental subjects, the ubiquity of social networks suggest perfectly independent observations are unlikely.

development affects domestic democracy), because the units influence one another (i.e., interdependence/contagion of democracy), &/or due to clustering, spillovers, or interdependence in unobservables (culture, perhaps).[12] Moreover, whether by clustering, spillovers, or contagion, we can expect spatial (cross-unit) dependence to manifest across the entire substantive range of political science—intergovernmental diffusion of policies and institutions among nations or subnational jurisdictions (e.g., Graham et al. 2013); international diffusion of democracy (e.g., Starr 1991); parties', representatives', and citizens' votes and other behaviors in legislatures and elections (e.g., Kirkland 2011; Tam Cho & Fowler 2010; Baybeck & Huckfeldt 2002); interdependence in globalization studies (e.g., Simmons & Elkins 2004) and contextual/neighborhood effects in microbehavioral research (e.g., Huckfeldt & Sprague 1987); wars, coups, riots, civil wars, revolutions, terrorism (e.g., Buhaug & Gleditsch 2008)—and many more. Indeed, interdependence across units is a defining characteristic of the *social* sciences, where its study is prominent also in geography & environmental sciences; in regional, urban, & real-estate economics; in medicine, public health, & epidemiology; in education, psychology, sociology, & social-psychology; and beyond.

Spatial dependence, in short, is everywhere, empirically and substantively/theoretically. Applied researchers almost always, perhaps unknowingly, account for some clustering in regression models simply through the inclusion of exogenous covariates, which also cluster ubiquitously. We call this *clustering in observed covariates* and note that its corresponding model is nonspatial (NON): $y_{it}=\mathbf{x}_{it}\boldsymbol{\beta}+\varepsilon_{it}$.[13] Insofar as these spatially clustered $\mathbf{x}$ are omitted or are inadequate to account the full spatial dependence in the dependent variable, the remainder will manifest as spatially correlated errors, as anything omitted from the systematic component (mean function) is shunted to the residual component. As shown elsewhere (Franzese & Hays 2007, 2008*a*), left unaddressed, such spatial dependence risks inefficiency at best and typically bias as well.

Often, though, additional sources of spatial correlation—correlated unobservables, exogenous

[12]We sidestep here issues of spatial-unit aggregation, i.e. the *MAUP: Modifiable Areal Unit Problem* (Fotheringham & Wong 1991), which are similar to, but more complex than, the more-familiar issue of temporal granularity/aggregation affecting time-serial dependence (Stram & Wei 1986; Freeman 1989). Likewise, we do not emphasize crucial specification issues regarding $\mathbf{W}$, the matrix of relative connectivity or distance between the units, i.e. the network, by which spatial association manifests (see, e.g., Franzese & Hays 2008*b*; Neumayer & Plümper 2016).

[13]We assume linear-additive separable mean and stochastic component here solely for ease of exposition.

spillovers, &/or outcome interdependence—are also present. When other manifest sources are omitted, including spatially correlated $\mathbf{x}$ regressors not only fails to fully address spatial dependence, but can actually further compromise our understanding of the data-generating process. These included $\mathbf{x}$ have power against the unmodeled spatial processes, which biases their coefficient estimates following the familiar omitted-variable bias (OVB) formula and logic (Franzese & Hays 2007, 2008*a*). Accordingly, political scientists have increasingly sought to model these other spatial processes directly also, using the workhorse models of spatial econometrics—spatial-error model (SEM), spatially-lagged $\mathbf{x}$ model (SLX), and spatial-lag (of y) model (SAR)—each of which assumes and reflects a single additional source of cross-unit dependence—correlated unobservables, exogenous spillovers, and outcome interdependence, respectively—via an additional modeling device, the *spatial lag*, to bring 'neighboring' values of $\boldsymbol{\varepsilon}$, $\mathbf{x}$, or $\mathbf{y}$ into the model. A brief summary of these models will help establish concepts and notation which may be unfamiliar to some readers.

Each of these spatial models, and indeed any spatial analysis whatsoever, even merely measuring & testing spatial correlation, must begin with specifying the connectivity (or spatial-$w$eights) matrix, $\mathbf{W}$, a $N \times N$ matrix with elements w_{ij} reflecting the relative connection, tie, distance, or potential influence, from unit j to unit i. This (pre-)specification of $\mathbf{W}$ is primary to any spatial analysis (Neumayer & Plümper 2016), being essential for preliminary descriptives and diagnostics, model specification and estimation, and effects calculation. Any relational data (e.g., trade, alliances, joint membership) can undergird $\mathbf{W}$, and of course theory and substance should always be paramount in this indispensable foundational step of spatial analysis. Absent strong theory, though, researchers often use geographic proximity since geography correlates with so many other potential bases for interconnection: economic interchange, cultural and linguistic similarities, and flows of people and information, e.g., are all greater across borders than between more-distant states.[14] Different specifications of $\mathbf{W}$ allow researchers to study diverse empirical patterns and alternative substantive/theoretical bases of cross-unit relations.[15] The researcher defines the rel-

[14]Given uncertainty over the relevant ties/network, a Bayesian Model-Averaging approach to *estimating* $\mathbf{W}$ simultaneously with a model of its effect seems promising (Juhl 2020).

[15]While misspecified $\mathbf{W}$ will, of course, reduce the accuracy and power of spatial-association tests and measurements and spatial-model estimates, research has shown that the consequences of errors in $\mathbf{W}$ are often less severe than feared (LeSage & Pace 2014) and certainly better than ignoring spatial dependence outright (Betz et al. 2020).

evant concept of space and metric of distance for her application—again, geographic distance or contiguity is often convenient and powerful default, and will be ours here—and then usually *normalizes* this $\mathbf{W}$ in some manner to ease interpretation, reduce dependence on scale factors, ensure the invertibility of the spatial multiplier, etc. The most-common *row normalization*, dividing each w_{ij} by row-sum $\sum_j w_{ij}$, produces spatial lags equal to weighted-averages of $\mathbf{x}$ (as defined by $\mathbf{W}$) and thereby facilitates direct interpretation of the lag coefficient among other conveniences.[16,17] With $\mathbf{W}$ specified and normalized, it then pre-multiplies a vector—$\boldsymbol{\varepsilon}$, $\mathbf{x}$, or $\mathbf{y}$—to produce so-called spatial lags—$\mathbf{W}\boldsymbol{\varepsilon}$, $\mathbf{Wx}$, or $\mathbf{Wy}$, which are weighted (by $\mathbf{W}$) averages of ($\mathbf{W}$-defined) neighbors' errors, covariates, or outcomes—for use in preliminary measures & tests of spatial correlation (e.g., Moran's I), in specification & estimation of spatial models, and in interpretation of spatial effects.

Quickly reviewing the baseline spatial models: the spatial error model (SEM) assumes spatially autocorrelated residuals, which are orthogonal to the included regressors. As mentioned, a spatial-error process can arise from clustering, spillovers, or interdependence in unobserved or unmodeled, but orthogonal, factors, resulting in a non-spherical error variance-covariance matrix and consequently inefficient OLS estimators. In the democracy-development example, spatial error dependence may occur due to unmodeled country-specific determinants of democracy (e.g., cultural/historical legacies (Acemooglu et al. 2008)) or from heterogeneity across countries in the effect of development on democracy (i.e., *spatial heterogeneity*). Formally, the SEM model is:[18]

$$\mathbf{y} = \mathbf{x}\beta + \mathbf{u}, \text{ with } \mathbf{u} = \lambda \mathbf{W}\mathbf{u} + \boldsymbol{\varepsilon} \tag{1}$$

with $\mathbf{W}$ the $N \times N$ connectivity matrix with elements w_{ij} reflecting the relative connectivity from j to i, and λ the strength of spatial autocorrelation propagated in this predetermined pattern, $\mathbf{W}$.

Next, cross-unit spillovers or externalities in exogenous observed factors (regressors, $\mathbf{x}$) can also produce spatial dependence in outcomes. In our democracy-development example, exogenous

[16] Some other common normalizations include spectral or min-max, which have other convenient properties.

[17] Neumayer & Plümper (2016) further discuss $\mathbf{W}$ specification and normalization issues, the most important being that the choice of normalization, or not to normalize equally as well, affects the substantive interpretation of the lag variable and coefficient.

[18] This SEM assumes spatial-autoregressive errors; moving-average (spillover) or spatial-hierarchical (clustered) versions also exist. The distinctions are not easily discerned empirically, being unobserved processes in unobserved factors. Fortunately, the distinctions are also immaterial to *effects* as we've defined them here.

spillovers occur if economic development in a country influences, not only its own democracy, but that of neighboring countries as well, perhaps via development spurring the emergence of transnational advocacy networks as discussed in Keck et al. (1998). Alternatively, conflict or public health in neighboring countries, x_j, may influence probabilities of democratic emergence or stability at home, y_i. The spatial-lag $\mathbf{x}$ or SLX model captures exogenous spillovers like these:

$$\mathbf{y} = \mathbf{x}\beta + \mathbf{W}\mathbf{x}\theta + \boldsymbol{\varepsilon}. \tag{2}$$

Here, the spatial lag of regressor, $\mathbf{Wx}$, introduces neighboring (as per $\mathbf{W}$) values of $x_{j \neq i}$ into the model for y_i.[19] With $\mathbf{x}$ exogenous, $\mathbf{Wx}$ is too, so SLX models can be estimated efficiently by OLS, with $\hat{\theta}$ giving the magnitude of these exogenous spatial spillovers. Halleck Vega & Elhorst (2015) and, more recently for political science, Wimpy et al. (2021) offer further discussions of SLX.

Finally, where theory &/or substance indicate interdependence or contagion in outcomes, the increasingly widely-used SAR model is called for:

$$\mathbf{y} = \rho\mathbf{W}\mathbf{y} + \mathbf{x}\beta + \boldsymbol{\varepsilon}. \tag{3}$$

This SAR (spatial-lag y) model may be most familiar to readers, as it has quickly become the dominant model of applied spatial work in political science (and elsewhere). As previously noted, autoregressive processes like SAR are appropriate for interdependent/contagious processes. In the democracy-development example, Starr 1991's "Democratic Dominoes" notion implicates such spatial autoregression most directly: democracy is contagious; neighboring democracies *cause* democracy at home. Mechanisms for such causal contagion could be suasion, i.e. diplomacy and foreign policies, or demonstration effects: being surrounded by democracies could reveal much to domestic actors about the workings, prerequisites, benefits and costs of democracy (Elkink 2011).

The key substantive differences of spatial-autoregressive compared to the other processes are the aforementioned exponentially reverberating dynamic and steady-state effects. The key methodological difference is that the spatial-lag regressor, $\mathbf{Wy}$, being other units' outcomes, i.e. the endogenous dependent variable, is an endogenous regressor. Thus, consistent estimation of

[19]To simplify exposition, we use a single covariate and lag; the generalization to multiple covariates and lags is straightforward. Wimpy et al. (2021) discuss several advantages of this general SLX model.

SAR models requires instrumental variables (spatial two-stage least-squares or generalized method-of-moments) or systems maximum likelihood (spatial-ML). We suspect SAR's popularity among these single-source spatial models owes, one, to its substantive resonance in political science, where outcomes are often social and/or strategic behaviors wherein some units' outcomes/choices directly influencing others' outcomes/choices is endemic; and, two, to how the other two single-source models imply that clustering or spillovers occur only in observed/modeled or only in unobserved/unmodeled components, which seems generally less plausible than that dependence would operate in both as in SAR. (SAR does impose equal, autoregressive processes in observed & unobserved components, though, which may seem restrictive.)

In any case, these single-source models can be combined in whatever pairs[20] may be substantively/theoretically implicated. If, e.g., one expected spillovers in observed covariates (SLX) and in unobserved features (SEM), but not necessarily to the same extent or autoregressively as SAR implies, this SLX+SEM combination gives the so-called Spatial Durbin Errror Model (SDEM):

$$\mathbf{y} = \mathbf{x}\beta + \mathbf{W}\mathbf{x}\theta + \mathbf{u}, \text{ with } \mathbf{u} = \lambda\mathbf{W}\mathbf{u} + \boldsymbol{\varepsilon}. \tag{4}$$

These multi-source models are advantageous in that they allow researchers to simultaneously account for alternative spatial processes (here exogenous spatial spillovers and spatial error autocorrelation). This is significant because spatial-model specifications often have power against 'incorrect' alternative spatial processes: SAR, SLX, or SEM lag-coefficients or tests will 'pick up' unmodeled SLX, SEM, or SAR processes.[21] As a consequence, modeling one source of spatial dependence (e.g., SAR) while neglecting others (e.g., SEM) risks inaccurate (typically: inflated) estimates of the included dependence parameter. As a consequence, researchers are advised to condition on these potential alternative processes when performing diagnostic tests (Anselin et al. 1996*a*) or specifying their empirical models (Cook et al. 2020). Below we build on this, demonstrating that in TSCS data not only do different spatial models have power against alternative spatial processes, but alternative *temporal* processes as well. This motivates our suggested STADL

[20]Even the three-source model is estimable, albeit with great fragility, being identified by functional-form differences across the lag-y, x, ε processes (Elhorst 2014; Cook et al. 2020).

[21]Cook et al. (2015), Rüttenauer (2019), and Cook et al. (2020) explore the similarities and difference among these alternative specifications in the purely spatial (cross-sectional) context.

model, which combines multiple dependence sources across both spatial and temporal dimensions.

2.2 Temporal Dependence

Many readers may be more familiar with the time-series analogs to the spatial processes/models just described, owing to discussions in Keele & Kelly (2006) and elsewhere, so we will be brief here. As with space, temporal dependence or serial correlation may arise from four sources: y_t may correlate with y_{t-1} simply because exogenous covariates $\mathbf{x}$ correlate over time, because unobserved/unmodeled factors ε exhibit serial correlation, because past values of $\mathbf{x}_{t-s}$ have lagged effects on current outcomes y_t, &/or because past outcomes y_{t-s} themselves continue to shape current outcomes y_t, i.e. outcomes are persistent, exhibit inertia. Also as with space, these alternative sources correspond to distinct substantive/theoretical processes and model specifications. Analogous to the nonspatial model is the (identical) static model, $y_t{=}x_t\beta{+}\varepsilon_t$, corresponding in our substantive example to democracy exhibiting serial correlation simply because exogenous covariate development does. The SEM analog is the familiar serially correlated errors (SCE) model, $y_t{=}x_t\beta{+}u_t$, with $u_t{=}\delta u_{t-1}{+}\varepsilon_t$,[22] which reflects persistence in unobserved/unmodeled factors, such as cultural-historical legacies, perhaps. The finite distributed-lag (FDL) model, $y_t{=}x_t\beta{+}x_{t-1}\gamma{+}\varepsilon_t$, corresponds to the SLX model; substantively: past realizations of development directly affect present democracy, i.e. effects of x occur with a lag. Perhaps development spurs long-term sociocultural changes whose impact materializes later. Finally, in the temporal autoregressive (in y) process, i.e. the lagged-dependent-variable (LDV) model, $y_t{=}\phi y_{t-1}{+}x_t\beta{+}\varepsilon_t$, past democracy directly influences present democracy, i.e. a persistent or inertial process, which in this substantive case may reflect democratic institutionalization wherein experience with democracy itself yields increasingly entrenched or consolidated democracy (Alexander 2001; Diamond 1994). Again in parallel with the spatial context, effects (of x on y) in the static or SCE model are static: $\frac{dy_t}{dx_t}{=}\beta$ and $\frac{dy_s}{dx_t}{=}0\ \forall\ s{\neq}t$; whereas effects are dynamic in the FDL and LDV models, decaying discretely and persisting only to the lag-length order in FDL models but persisting infinitely

[22] As before, we continue here with first-order, i.e. one-period, lags for expositional simplicity (see also note 9).

with exponential/geometric decay, implying long-run steady-state (LRSS) multipliers, $\frac{1}{1-\phi}$, and cumulative LRSS effects, $\frac{1}{1-\phi} \cdot dx \cdot \beta$, in the autoregressive LDV.[23]

2.3 Spatiotemporal Dependence

With readers (re)familiarized with the base temporal and spatial dependence models/processes, we turn next to illustrating how these spatial and temporal dependencies are *necessarily* related. Start with the simple static/nonspatial linear-regression model, now indexed by unit i and time t:

$$y_{it} = \beta_0 + \beta_1 x_{it} + u_{it}, \tag{5}$$

except here assume that some residual dependence may result from omitted $y_{i,t-1}$, $y_{j,t}$'s,[24] or both:

$$u_{it} = \phi_y y_{i,t-1} + \rho_y \sum_{n=1}^{N} w_{ij} y_{j \neq i,t} + \varepsilon_{it}, \text{ with } \varepsilon_{it} \sim N(0, \sigma^2). \tag{6}$$

Furthermore, let x be stochastic, exogenous, and likewise follow its own spatiotemporal process:

$$x_{it} = \phi_x x_{i,t-1} + \rho_x \sum_{n=1}^{N} w_{ij} x_{j \neq i,t} + e_{it}, \text{ with } e_{it} \sim N(0, \sigma^2). \tag{7}$$

Given all other standard regression assumptions, we now walk through the relationship between spatial and temporal dependence (also depicted visually in Figures 2-5).

$$x \xrightarrow{\quad\quad\beta\quad\quad} y$$

Figure 2: Static Relationship

First, obviously, restricting ϕ_y=0 and ρ_y=0 produces *i.i.d.* residuals u_{it}, so the nonspatial, static equation (5) depicted in Figure 2 fully accurately models the relationship of x to y. Relaxing one restriction, say $\phi_y \neq 0$, but keeping the other, ρ_y=0, induces time-serial dependence in the residuals u, which biases $\hat{\beta}$ in the static model if $\phi_x \neq 0$. This situation, depicted in Figure 3, is textbook omitted-variable bias (OVB)—with $\text{Cov}(x, y_{t-1})$ increasing in ρ_x—and is easily remedied by including time-lagged y (LDV model) as commonly done. Similarly, freeing $\rho_y \neq 0$ while keeping ϕ_y=0 also threatens OVB in the static model. Again, OVB arises if x has dependence in the same

[23] Also analogously (see note 25), the question of the "effect of x on y" in temporally dynamic contexts requires more precise statement of both the hypothetical/counterfactual, dx, and the effect, dy, refining to specify dx *when*, in what period(s) is x 'shocked', and dy *when*, in what period(s) do we want to know the response of y thereto?

[24] $\sum_{n=1}^{N} w_{ij} y_{j \neq i,t}$ is the scalar representation of the spatial lag presented above in matrix form, i.e. $\mathbf{Wy}$.

dimension as y, here if $\rho_x \neq 0$ as depicted in Figure 4, so that $\text{Cov}(x, y_j) \neq 0$, and the simple remedy, increasingly common in applied work, adds spatial-lag y to form the spatially dynamic SAR model.

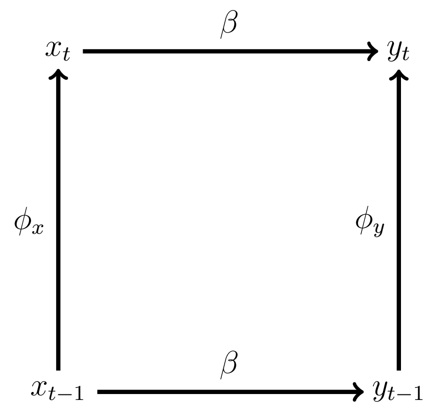


Figure 3: Time-serial Dependence

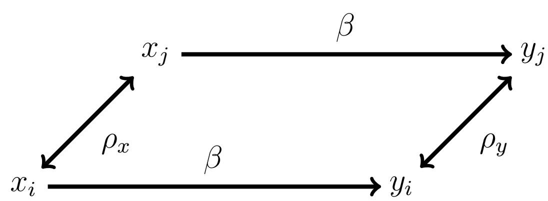


Figure 4: Cross-Sectional Dependence

This is all familiar: with single-dimensional dependence, purely cross-sectional spatial or time-serial modeling suffices. However, if *both* $\phi_y \neq 0$ *and* $\rho_y \neq 0$ as in Figure 5, i.e. with *both* temporal and spatial dependence present, researchers must model dependence in *both* dimensions adequately. Omitting/mismodeling spatial dynamics, e.g., will leave residual time-serial correlation because the omitted/mismodeled spatial-lag y_{jt} is serially correlated to $y_{j,t-1}$ which in turn exhibits that same omitted/mismodeled spatial relation to the included time-lag $y_{i,t-1}$. Symmetrically, failing to model temporal dynamics adequately will leave spatial autocorrelation, as the missed aspect of the past, $y_{i,t-1}$, has the same spatial relation to $y_{j,t-1}$ as does y_{it} to the included spatial-lag, y_{jt}.

We can prove this, that spatiotemporal dependence causes bias (OVB) when only one of spatial or temporal dependence is modeled, using the first-order spatiotemporal-lag model 20 (also depicted in Figure 5). If the truth is $\mathbf{y}_t = \beta \mathbf{x}_t + \rho \mathbf{W} \mathbf{y}_t + \phi \mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t$, but one estimates SAR (omitting $\phi \mathbf{y}_{t-1}$) or LDV (omitting $\rho \mathbf{W} \mathbf{y}_t$), then OVB arises if $\rho \phi \text{Cov}(\mathbf{W} \mathbf{y}_t, \mathbf{y}_{t-1}) \neq 0$. This covariance is necessarily nonzero because spatial dependence implies $\mathbf{W y_t} \longleftrightarrow \mathbf{y_t}$ and temporal dependence

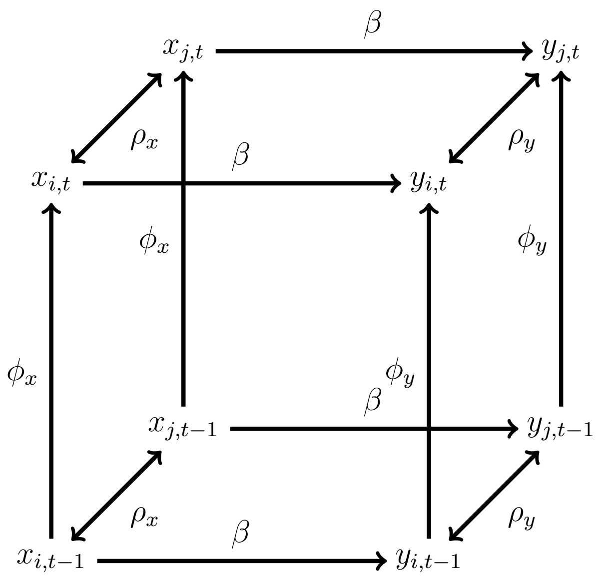


Figure 5: Space-Time Dependence

implies $\mathbf{y_{t-1}}\longrightarrow\mathbf{y_t}$, so $\mathbf{y_{t-1}}\longrightarrow\mathbf{y_t}\longleftrightarrow\mathbf{Wy_t}$ and $\text{Cov}(\mathbf{Wy}_t,\mathbf{y}_{t-1})=\text{Cov}(f(\mathbf{y}_t),\mathbf{y}_{t-1})\neq 0$. To see the sign and magnitude of these OVB, consider Achen (2000)'s derivation of the biases in $\hat{\phi}_y$ and $\hat{\beta}$ in the LDV model when additional, unmodeled dynamics ϕ_e remain in the disturbance term:

$$\text{plim}\,\hat{\phi}_y = \phi_y + \left[\frac{\phi_e \sigma^2}{(1-\phi_e\phi_y)s^2}\right], \tag{8}$$

$$\text{plim}\,\hat{\beta} = \left[1 - \frac{\phi_x g}{1-\phi_x\phi_y}\right]\beta, \tag{9}$$

where $s^2=\sigma^2_{Y_{t-1},X}$ and $g=\text{plim}(\hat{\phi}_y)-\phi_y$ (see also Keele & Kelly 2006). Achen (2000) notes that any $\phi_e>0$ inflates $\hat{\phi}_y$ and attenuates $\hat{\beta}$ estimates. Notice, as just proven, that any unmodeled spatial dependence *necessarily* produces precisely these conditions, as $y_{i,t}=\phi_y y_{i,t-1}+x_{i,t}\beta+u_{i,t} \implies y_{j,t}=\phi_y y_{j,t-1}+x_{j,t}\beta+u_{j,t}$, therefore any $\rho_y\neq 0$ produces $\phi_e>0$ and 'Achen's LDV-bias'. Following now the simple OVB logic: omission or underestimation of ρ_y induces primarily overestimation (inflation bias) of ϕ_y, being the coefficient on the included regressor most related to the omitted/mismeasured $\mathbf{Wy}$, and that in turn induces compensatory deflation bias of β_x. Thus, even if Stimson (1985)'s 'inherent' (temporal) autocorrelation is accurately modeled, misspecification in the spatial dynamics sets off a chain of biases: the primary attenuation (underestimation or 0 if omitted) of ρ_y, induces overestimation (inflation bias) of ϕ_y, which induces attenuation (deflation, underestimation) of β_x, and of course any related causal-inference tests are biased thereby as well.

As a result of all these parameter-estimate biases, the dynamic and total causal effects of x on y are misestimated too: initial 'impulses' (β) from x to y underestimated, spatiotemporal dynamics misconstrued to 'too-persistent' if spatial dependence omitted or relatively mismodeled or 'too-contagious' if temporal dependence omitted (rare) or relatively mismodeled (more common), and so long-run steady-state effect estimates will be biased also.

Given that inadequate address of spatiotemporal dependence will bias inferential tests and estimates of coefficients, dynamics, and steady-state effects, even researchers for whom these dynamics and dependencies are nuisance cannot neglect their careful attention. Furthermore, these biases induced by relative neglect of spatial or, less commonly, temporal dependence are of central substantive-theoretical importance as well. In our *development-and-democracy* terms, relative inadequacy in addressing spatial dependence—inadequate account in the model that, and by what process, democracy clusters—yields estimates that imply inaccurately greater temporal persistence of democracy, e.g. an overestimate of democratic-institutionalization and -consolidation effects. If democratic persistence derives from a temporally autoregressive process as such arguments imply, this overestimated temporal dependence will mean slower geometric decay of, and larger long-run-steady-state multipliers on, other covariates' effects on democracy, which covariates, such as development, will in turn have smaller immediate-impact estimates, i.e., smaller $\hat{\beta}_x$. Moreover, along with these misestimated dynamic and steady-state effects, the biased $\hat{\beta}_x$ mean that hypothesis tests (inferences) about the (causal) effects of x on y will be biased as well, likely increasing Type II error (lack of power, failure to reject when should).[25] These biases arise because, in a TSCS analysis with temporal dependence modeled but spatial dependence excluded, for instance, what among the included factors looks most like the omitted 'today's democracy abroad'—say German democracy today ($y_{j,t}$) as omitted explanator of French democracy today ($y_{i,t}$)—is 'yesterday's democracy at home', i.e. French democracy yesterday (*time*-lagged $y_{i,t-1}$). Intuitively, as shown mathematically and diagrammatically above, because, and insofar as, 'Germany yesterday' relates to 'France yesterday'—spatial dependence is present—and 'Germany yesterday' relates to

[25] Notice also that, in spatiotemporally autoregressive contexts, the usual statement of the causal estimand, 'the effect of x on y' is itself underspecified, because, for the question to be fully enunciated given spatiotemporal interdependence, we need to ask about 'the effect of x *when and where* on y *when and where*'.

'Germany today'—temporal dependence is also present, i.e., with both spatial interdependence and temporal dependence present, the omitted 'Germany today' relates to the included 'France yesterday'. Of course, all of the analogous holds also in the other direction, regarding the (rarer in applied work) omission or relatively inadequate address of temporal dependence.[26]

Applied researchers also commonly deploy unit or period fixed-effects to 'account for' spatial or temporal dependence. Unit or period dummies (or random effects) do address particular forms of spatiotemporal dependence (Elhorst 2014), but often fail to fully characterize the patterns of spatiotemporal dependence found in TSCS data. Unit indicators absorb long-run, fixed or constant, spatial clustering in outcomes, plus any other time-invariant unobserved/unmodeled unit-specific factors. However, these captured 'effects' are additive, mean-shifts, time-invariant *clustering*, and *not* autoregressive or distributed-lag in form. Unit-specific effects also cannot account time-varying unobserved/unmodeled effects (such as evolving spatially clustered sociocultural or institutional factors). Analogously, period fixed-effects/time-dummies account for 'global' shocks: spatially-invariant, uniform common across all units, fixed, additive mean-shifts, and so cannot account autoregressive or distributed-lag processes, or unit or regional variation in clustered additive shocks (such as influences diffusing among, or additive unobserved characteristics of, members of regional organizations).

Finally, given the substantively and statistically critical importance of adequate address of spatiotemporal dependence, researchers will want to conduct appropriate and effective specification testing.[27] In principle, one can conduct specification searches from 'specific-to-general', starting with sparse spatiotemporal models and testing, using Lagrange-Multiplier (LM) tests, whether to add spatiotemporal-lag terms, or 'general-to-specific', starting with a more-general specification and testing, by Wald (t or F) or loss-of-fit (ΔR^2 or likelihood-ratio) tests, whether specific

[26] In practice, given the typically great strength of time-lags as predictors compared to other regressors, omitted spatial factors' relation to included temporal factors is usually by-far the strongest of the OVB formula's partial correlations, meaning inadequate address of spatial dependence induces largest inflationary biases on the temporal-dependence parameters and secondary induced biases in other covariate coefficients. Conversely, included (better specified) spatial dependence being typically considerably weaker than omitted (more-poorly specified) temporal dependence, and the temporal persistence of other exogenous covariates being likewise stronger than their spatial association, the OVB biases tend to be more-evenly distributed across included parameters.

[27] And sensitivity analyses (see, e.g., Neumayer & Plümper 2017).

spatiotemporal-lag terms may safely be omitted (Hendry 1995). In this present context, though, we know LM tests of underspecified models will 'have power against incorrect alternatives' (Anselin 1988): e.g., rejecting LDV in favor of adding SAR when the actual missing spatial process is SEM or SLX, or rejecting SAR in favor of adding SEM when it is the temporal-dependence process that is missing/poorly specified.[28] Instead, we suggest the (first-order) Spatio-Temporal Autoregressive-Distributed-Lag (STADL) model, as a convenient and effective more-general starting point (i.e., adequately general and encompassing for most TSCS applications in political science).

In summary, as we will further demonstrate by simulation and in applications re-analyses below, TSCS analyses of, e.g., the *democracy-and-development* proposition that relatively neglect spatial (temporal) dependence will estimate greater temporal persistence (spatial dependence) than actually present, and correspondingly misestimate *spatiotemporal* dynamic and cumulative effects, and so yield biased tests and erroneous inferences regarding substantive-theoretical propositions. The more-general STADL model offers effective alternative for applied TSCS analyses.

3 The STADL Model

The workhorse cross-sectional and time-serial models from spatial and time-series econometrics were introduced above. To review compactly, the baseline spatial-econometric models correspond to the different potential sources for observed spatial association: nonspatial models (NON) for spatially clustered exogenous covariates (including fixed-effects), spatial error (SEM) for clustering in unobservables, spatially lagged covariates (SLX) for exogenous spillovers/externalities, and

[28]LM tests can be adjusted (using cross-partial gradients of the fuller-specification likelihood) to prevent rejection against specific incorrect alternatives, but these 'robust LM tests' (Anselin et al. 1996*b*) as yet exist for very few combinations of spatiotemporal processes.

spatial-lag/spatial-autoregressive (SAR) models for endogenous contagion/interdependence:[29]

$$\text{Clustered Covariates} = NON : \mathbf{y}_t = \mathbf{x}_t\beta + \boldsymbol{\varepsilon}_t, \text{ with } \mathbf{x}_t \text{ spatially correlated} \tag{10}$$

$$\text{Clustered Unobservables} = SEM : \mathbf{y}_t = \mathbf{x}_t\beta + \mathbf{u}_t, \text{ with } \mathbf{u}_t = \lambda\mathbf{W}\mathbf{u}_t + \boldsymbol{\varepsilon}_t \tag{11}$$

$$\text{Spillovers/Externalities} = SLX : \mathbf{y}_t = \mathbf{x}_t\beta + \mathbf{W}\mathbf{x}_t\theta + \boldsymbol{\varepsilon}_t \tag{12}$$

$$\text{Interdependence/Contagion} = SAR : \mathbf{y}_t = \rho\mathbf{W}\mathbf{y}_t + \mathbf{x}_t\beta + \boldsymbol{\varepsilon}_t \tag{13}$$

Notice, crucially, that 'the effect of x' differs importantly across these models. With clustered exogenous covariates (NON), $\frac{dy_{it}}{dx_{it}}{=}\beta$ (and $\frac{dy_{js}}{dx_{it}}{=}0\ \forall\ j{\neq}i, s{\neq}t$). Likewise with spatial dependence confined to the *orthogonal* unobserved component (SEM), the effect of x on y is merely $\frac{dy_{it}}{dx_{it}}{=}\beta$ (and $\frac{dy_{js}}{dx_{it}}{=}0\ \forall\ j{\neq}i, s{\neq}t$). In our substantive example, in both of these models/sources/processes: 'What happens in France stays in France' with respect to the effect of x on y. With exogenous externalities (SLX), i.e. with the spatial distributed-lag model/source/process, 'What happens in France spills over into Germany (and France's other first-order neighbors according to $\mathbf{W}$),' and the story ends there: $d\mathbf{y}{=}\mathbf{W}\cdot d\mathbf{x}_t\cdot\beta$. Notice that both the hypothetical/counterfactual $d\mathbf{x}_t$ and the effect, $d\mathbf{y}_t$ are vectors, not scalars; with spatial spillovers, the effect of x differs depending on which units are 'shocked' and these effects manifest not only in y_i of the shocked unit/s but also in its/their (first-order) neighbors as defined by $\mathbf{W}$.[30] In the spatial-autoregressive model that corresponds to interdependent/contagious contexts, 'what happens in France influences Germany & France's other neighbors, which in turn influence their neighbors, including France, which in turn influence those neighbors' neighbors' neighbors, including Germany again, and so on,' with the effect of any $d\mathbf{x}_t$ on $\mathbf{y}_t$ reverberating outward and back thusly in an exponentiating series:

$$\begin{aligned} d\mathbf{y}_t &= (\underbrace{\mathbf{I}}_{\text{self}} + \underbrace{\rho\mathbf{W}}_{\text{neighbors}} + \underbrace{\rho^2\mathbf{W}^2}_{\text{neighbors' neighbors}} + \underbrace{\rho^3\mathbf{W}^3}_{\text{neighs' neighs' neighs}} + \underbrace{\rho^4\mathbf{W}^4}_{\text{neighbors}^4} + \ldots)\cdot d\mathbf{x}_t\cdot\beta \\ &= \left(\sum_{m=0}^{\infty}\rho^m\mathbf{W}^m\right)\cdot d\mathbf{x}_t\cdot\beta = \underbrace{(\mathbf{I}-\rho\mathbf{W})^{-1}}_{\text{spatial multiplier}}\cdot\underbrace{d\mathbf{x}_t}_{\text{shock}}\cdot\underbrace{\beta}_{\text{impulse}} \end{aligned} \tag{14}$$

Again, insofar as researchers misspecify (or omit) the spatial-dependence process, say $\mathbf{Wy}$, ρ is underestimated ($\rho{=}0$ if omitted), and the OVB formula and intuition implies inflated $\boldsymbol{\beta}$ estimates,

[29] The vectors in these equations are $N{\times}1$; the matrix $\mathbf{W}$ is $N{\times}N$.

[30] Whitten et al. (2021) discuss how higher-order SLX models, i.e. powers of $\mathbf{W}$, capture neighbor-of-neighbor effects, etc.

with those OVBs distributed proportionately to the $\mathbf{x}$'s partial association with the misspecified/omitted $\mathbf{Wy}$, meaning larger induced biases will accrue to the x's with spatial clustering more similar to that implied by $\mathbf{W}$.

The time-series analogs, also first-order, are compactly expressed using the lag operator, $\mathbf{L}^s\mathbf{y}_t \equiv \mathbf{y}_{t-s}$, as the serially correlated errors (SCE), finite distributed lag (FDL), and lagged dependent variable (LDV) models, along with the static model (StM) with serially correlated exogenous covariates (including time-period fixed-effects, and identically parallel to the nonspatial model):[31]

$$StM : \mathbf{y}_t = \mathbf{x}_t\beta + \mathbf{u}_t, \text{ with } \mathbf{x}_t \text{ serially correlated} \tag{15}$$

$$SCE : \mathbf{y}_t = \mathbf{x}_t\beta + \mathbf{u}_t, \text{ with } \mathbf{u}_t = \delta\mathbf{L}\mathbf{u}_t + \boldsymbol{\varepsilon}_t \tag{16}$$

$$FDL : \mathbf{y}_t = \mathbf{x}_t\beta + \mathbf{L}\mathbf{x}_t\gamma + \boldsymbol{\varepsilon}_t \tag{17}$$

$$LDV : \mathbf{y}_t = \phi\mathbf{L}\mathbf{y}_t + \mathbf{x}_t\beta + \boldsymbol{\varepsilon}_t \tag{18}$$

Notice again the dynamics, or lack thereof, of the effects of x on y in these time-series models. In the static and serially correlated errors models, the effect of x_t is confined to y_t, there are no temporal dynamics: $\frac{dy_t}{dx_t}{=}\beta$ and $\frac{dy_s}{dx_t}{=}0 \ \forall \ s{\neq}t$. In distributed-lag or autoregressive processes, contrarily, and again in parallel to the spatial cases, we need first specify *dx when* and expand our question about the effect on *y when.* In the distributed-lag case, the effects of x simply spill forward the number of periods equal to the lag-order, p, $d\mathbf{y}_t{=}\mathbf{L} \cdot d\mathbf{x}_t \cdot \beta$, and are completely dissipated beyond that: $\frac{d\mathbf{y}_{t+s}}{d\mathbf{x}_t}{=}0 \ \forall \ s{>}p$. Temporally autoregressive processes, finally, imply exponentiating (geometric) decay for 'temporary shocks', or decaying accumulation for 'permanent shocks', of 'long-run steady-state' effects going forward infinitely in time, like so:

$$\underbrace{dy_\infty}_{\substack{\text{LRSS}\\\text{response}}} = \underbrace{\beta dx}_{\text{period 0}} + \underbrace{\rho\beta dx}_{\text{period 1}} + \underbrace{\rho^2\beta dx}_{\text{period 2}} + \underbrace{\rho^3\beta dx}_{\text{period 3}} + ... = \underbrace{\sum_{s=0}^{\infty}\rho^s\beta dx}_{\text{if } 0<\rho<1,\ \Rightarrow} = \underbrace{\left(\frac{1}{1-\rho}\right)}_{\text{LR multiplier}} \times \underbrace{\beta}_{\text{impulse}} \times \underbrace{dx}_{\substack{\text{perm.}\\\text{shock}}} \tag{19}$$

It can be intuited from these differing expressions of the '(causal) effects of x on y' implied by the range of possible spatial and temporal processes that omissions or misspecifications of either temporal or spatial dependence, given that they will induce biased estimates of the other

[31] The standard autoregressive distributed lag notation ADL(p, q) signifies time-lagged y of order p and time-lagged x of order q, which given linear additivity suffices to give the lag-ε model as well.

dependence-process' parameters and covariates' coefficients, will yield consequentially inaccurate tests and estimates of the substantive (causal) effects of interest.

Given this critical substantive and statistical importance of allowing the estimation model to express the spatiotemporal dependence inherent to TSCS data in the manner it manifests, we suggest to combine these models in a Spatio-Temporal Autoregressive Distributed Lag (STADL) model of order $(sy^0, sx^0, se^0; ty^1, tx^1, te^1)$, where the s or t indicate spatial or temporal lag, the y, x, e indicate which terms are lagged, and the superscript indicates the *temporal* order of the lag, s^0 for contemporaneous spatial lags, e.g.[32] We recommend including in parentheses only the terms actually used; the STADL(sy^0, ty^1), e.g., indicates the first-order spatiotemporal-lag model that has become somewhat common most recently:

$$\mathbf{y}_t = \rho\mathbf{W}\mathbf{y}_t + \phi\mathbf{L}\mathbf{y}_t + \mathbf{x}_t\beta + \boldsymbol{\varepsilon}_t, \tag{20}$$

while the general version of the STADL$(ty^p, tx^q, te^r, sy^P, sx^Q, sx^R)$ is

$$\mathbf{M}\mathbf{y}_t = \mathbf{F}\mathbf{x}_t + \mathbf{A}\boldsymbol{\varepsilon}_t, \tag{21a}$$

$$\mathbf{M} \equiv \left(\mathbf{I} - \phi_1\mathbf{L} - ... - \phi_p\mathbf{L}^p - \rho_0\mathbf{W} - ... - \rho_{P-1}\mathbf{W}^{P-1}\right), \tag{21b}$$

$$\mathbf{F} \equiv \left(\mathbf{I}\beta + \mathbf{L}\gamma_1 + ... + \mathbf{L}^q\gamma_q + \mathbf{W}\theta_0 + ... + \mathbf{W}^{Q-1}\theta_{Q-1}\right), \tag{21c}$$

$$\mathbf{A} \equiv \left(\mathbf{I} - \delta_1\mathbf{L} - ... - \delta_r\mathbf{L}^r - \lambda_0\mathbf{W} - ... - \lambda_{R-1}\mathbf{W}^{R-1}\right)^{-1}. \tag{21d}$$

where $\mathbf{M}$, $\mathbf{F}$, $\mathbf{A}$ are the space-time filters of the outcome, predictors, and residuals, respectively.[33]

We express a first-order STADL conveniently for interpretation of spatiotemporal effects as:

$$\mathbf{y} = \phi\mathbf{L}\mathbf{y} + \rho\mathbf{W}\mathbf{y} + \mathbf{x}\beta + \mathbf{L}\mathbf{x}\gamma + \mathbf{W}\mathbf{x}\theta + (\mathbf{I} - \delta\mathbf{L} - \lambda\mathbf{W})^{-1}\boldsymbol{\varepsilon}, \tag{22a}$$

$$\mathbf{y} = (\mathbf{I} - \phi\mathbf{L} - \rho\mathbf{W})^{-1}\left(\mathbf{x}\beta + \mathbf{L}\mathbf{x}\gamma + \mathbf{W}\mathbf{x}\theta + (\mathbf{I} - \delta\mathbf{L} - \lambda\mathbf{W})^{-1}\boldsymbol{\varepsilon}\right). \tag{22b}$$

where $\mathbf{I}$,$\mathbf{L}$, and $\mathbf{W}$ are $NT \times NT$ matrices; $\mathbf{y}$,$\mathbf{x}$,and $\boldsymbol{\varepsilon}$ are $NT \times 1$ vectors; and $\mathbf{L}$ creates a one-period

[32] For multiple spatial-weights matrices, $\mathbf{W}$, the s can be subscripted numerically or mnemonically, likewise in cases where only some regressors $\mathbf{X}$ are lagged. Researchers writing for audiences more-familiar with ADL and/or spatial notation, can use SAR+ADL(p,q) for instance.

[33] Although not a focus here, the STADL model can also easily incorporate recursive spatial processes (Anselin 2001) via time-lagged spatial lags (Drolc et al. 2019). Like $y_{i,t-1}$ or $\mathbf{W}\mathbf{x}$, time-lagged spatial-lags (TLSL) are predetermined in the system of equations, meaning they can be treated as exogenous regressors. For interpretation, in the spatiotemporal dynamic effects, the $\mathbf{L}$ matrix requires modification ($\mathbf{W}$ placed around the ones on the lower-block-minor diagonals).

time-lag of variables it premultiplies.[34] Differentiation of Equation 22 by $\mathbf{x}$ tracks responses over time across all N units to some series of hypothetical/counterfactual *shocks* in N units over T periods, $d\mathbf{X}$, an $NT{\times}N$ matrix of shocks in each unit-period, (n, t):[35]

$$d\mathbf{Y} = (\mathbf{I} - \phi\mathbf{L} - \rho\mathbf{W})^{-\mathbf{1}}(\mathbf{I}\beta + \mathbf{L}\gamma + \mathbf{W}\theta) \cdot d\mathbf{X}. \tag{23}$$

$d\mathbf{Y}$ gives in each column the response across all N units period-by-period to the 'shock' that unit experiences given in $d\mathbf{X}$. Recall that in spatiotemporal analyses, one must specified which units are shocked (experience the hypothetical/counterfactual) and when—that's the 'treatment'—and, correspondingly, the responses ('effects') will be in all units over all time-periods. In time-series, one must specify *dx when*, and the default shocks are called *temporary*, a one-period shock—$dx{=}{+}1$ in period $t0$ and $dx{=}0$ else—and *permanent*—$dx{=}{+}1$ in all periods. In spatial analysis, one must specify *dx where*, and the analogous defaults are $dx{=}{+}1$ in one unit and $dx{=}{+}1$ in all units. In space-time, the spatial and temporal defaults are combined to produce four; unit-i or all units $\times$ 1-period or permanently. Notice the ambiguity surrounding comparable *treatments* and *effects* in static versus in temporally, spatially, and spatiotemporally dynamic models/processes: in static/nonspatial models, x's effects incur exclusively in the unit-time shocked; in spatiotemporal models, x_{it} has effects also in units $j{\neq}i, s{\neq}t$.[36] Equation 23 gives the responses in $d\mathbf{Y}$ column by column to the shock given in that column of $d\mathbf{X}$. So, $d\mathbf{X}$ for the own-unit shock in period t is an $N{\times}N$ identity matrix (1 in diagonal elements (i, i), 0 else), and that $N{\times}N$ $\mathbf{I}$ occurs only in the first block of $d\mathbf{X}$ for the temporary shock, and repeats for all periods for the permanent. The all-units shock is a column of 1s, so every-unit/all-units shocked is an $N{\times}N$ matrix of all 1s, again: only in the first $N{\times}N$ of $d\mathbf{X}$ for temporary, repeated for all periods for permanent.

Long-run steady-state (LRSS) responses in all N units to some *permanent* $N{\times}1$ set of shocks,

[34] Spatiotemporal TSCS analyses order the data as all N units in period 1, all N units in period 2, ... $\mathbf{L}$'s $N{\times}N$ first block has all-0 elements, reflecting the omitted N first-period observations, all other elements are 0 too, except the diagonal of the lower first block minor (the $N{\times}N$ blocks immediately below the $N{\times}N$ prime block diagonal), those lower-block-minor diagonal elements are all 1.

[35] The dimensions in 23 are: $d\mathbf{Y}$,$d\mathbf{X}$ = $NT{\times}N$, and $\mathbf{I}$,$\mathbf{L}$,$\mathbf{W}$ = $NT{\times}NT$.

[36] In comparison to the static case, and to empirical realism, one-unit or one-period $d\mathbf{X}$ (radically) understates the counterfactual because the spatiotemporal model rightly allocates the total impact of $d\mathbf{X}$ on $d\mathbf{Y}$ across space and time. From this view, all-unit permanent $d\mathbf{X}$ exaggerates a realistic $d\mathbf{X}$, and static-model dx. Perhaps most realistic (and what static-model estimates would be approximating with bias due to misspecifcation) would be $d\mathbf{X}$ that followed the empirical spatiotemporal pattern.

$d\mathbf{x}$, is found by returning to (22a), setting y_{t-1}=y_t and x_{t-1}=x_t by definition of LRSS, to obtain:[37]

$$d\mathbf{y} = (\mathbf{I} - \phi\mathbf{I} - \rho\mathbf{W})^{-\mathbf{1}}(\mathbf{I}\beta + \mathbf{I}\gamma + \mathbf{W}\theta) \cdot d\mathbf{x}. \tag{24}$$

Note the shock/hypothetical/counterfactual $d\mathbf{x}$ in unit(s) i and response/effect $d\mathbf{y}$ in all N units.

STADL models can be estimated via (concentrated) maximum likelihood, or Bayesian methods, with likelihoods (posteriors) given in Elhorst (2001) (and LeSage & Pace 2009) and maximization detailed in Anselin (1988).[38] Even previous works that discuss TSCS data & spatiotemporal models have neither discussed or derived analytically as above, nor evaluated through simulation as next, the biases from omitting or mismodeling one of the dependence dimensions in estimates of the other dependence parameters, the covariate coefficients, and the dynamic and total effects.

4 Monte Carlo Analysis of Dynamic TSCS Models

Our Monte Carlo Analyses demonstrate that the biases shown analytically above are of substantively important magnitudes in spatiotemporal TSCS data with properties designed to be representative of common political-science application contexts. Given the combinatorically vast number of STADL-model variations—62 first-order models alone—we focus on evaluating the two currently most-widely used in political science: LDV and SAR, i.e., STADL(ty^1) and STADL(sy^0). LDV and SAR model performance under various forms of temporal *or* spatial dependence is well known, but less is known as-yet about the performance of either one-way model given *spatiotemporal* dependence in both dimensions. To explore SAR or LDV estimation performance given dependence also in the unmodeled (or, implicitly, mismodeled) other dimension, we generate data

[37] The dimensions in 24 are $d\mathbf{y}$,$d\mathbf{x}$ = $N\times1$, and $\mathbf{I}$,$\mathbf{W}$ = $N\times N$.

[38] Similar to the 3-source spatial model (see note 20), the 3-source temporal and STADL models are identified but frail when all 3 sources are included (i.e., the fully unrestricted STADL model). Given this, researchers will want to use design (Gibbons & Overman 2012) or theory (Cook et al. 2020) to restrict some spatial and temporal parameters *ex ante*. In Cook et al. (2020), we suggest that researchers should generally consider including terms capturing spillovers in the mean component (either $\mathbf{Wy}$ or $\mathbf{Wx}$) plus spatial error autocorrelation. Similarly, a time-series model including a time-lagged outcome and a correction for serially correlated errors would be robust to the concerns of Achen (2000). Taken together, we believe applied researchers with TSCS will be well served by including outcome lags ($\mathbf{Wy}$ and $\mathbf{Ly}$) *or* covariate lags ($\mathbf{Wx}$ and $\mathbf{Lx}$)—whichever is best motivated by their theory—and spatial and temporal error lags ($\mathbf{W}\boldsymbol{\varepsilon}$ and $\mathbf{L}\boldsymbol{\varepsilon}$).

from a STADL(sy^0, ty^1), i.e., the first-order spatiotemporal autoregressive model:[39]

$$\mathbf{y}_t = \phi_y \mathbf{y}_{t-1} + \rho_y \mathbf{W}\mathbf{y}_t + \mathbf{x}_t\beta + \boldsymbol{\varepsilon}_{\mathbf{y}}, \tag{25a}$$

$$\mathbf{x}_t = \phi_x \mathbf{x}_{t-1} + \rho_x \mathbf{W}\mathbf{x}_t + \boldsymbol{\varepsilon}_{\mathbf{x}}, \tag{25b}$$

with $\mathbf{x}_t$, $\boldsymbol{\varepsilon}_{\mathbf{y}}$, and $\boldsymbol{\varepsilon}_{\mathbf{x}}$ drawn independent standard-normal. To focus comparisons, we fix several conditions across simulation contexts. First, $N = 50$ and $T = 20$, giving a balanced panel with common sampling dimensions (e.g., U.S. states over 20 years). Second, we fix the parameters $\beta = 2$, $\phi_x = 0.6$, and $\rho_x = 0.3$. We vary for focal exploration the strength of temporal (ϕ_y) and spatial (ρ_y) dependence in the outcome $\mathbf{y}$ (further design details in the Appendix).

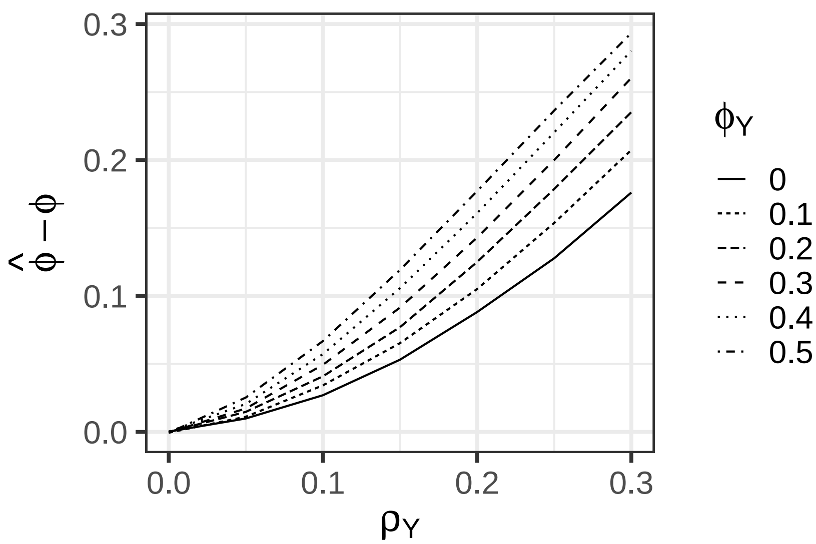


Figure 6: LDV Performance with Spatial Dependence — Bias in $\hat{\phi}_y$

[39] The Appendix also reports results using a STADL(se^0, te^1), i.e. the first-order spatiotemporal autoregressive *error* model, as the DGP. This allows us to explore the performance of commonly used *outcome*-lag models (LDV and SAR) and our STADL model under spatiotemporal *error* autocorrelation. Our results demonstrate that: *i)* the LDV and SAR models produced biased estimators under spatiotemporal error autocorrelation, *ii)* the STADL model, on the other hand, performs well under all conditions.

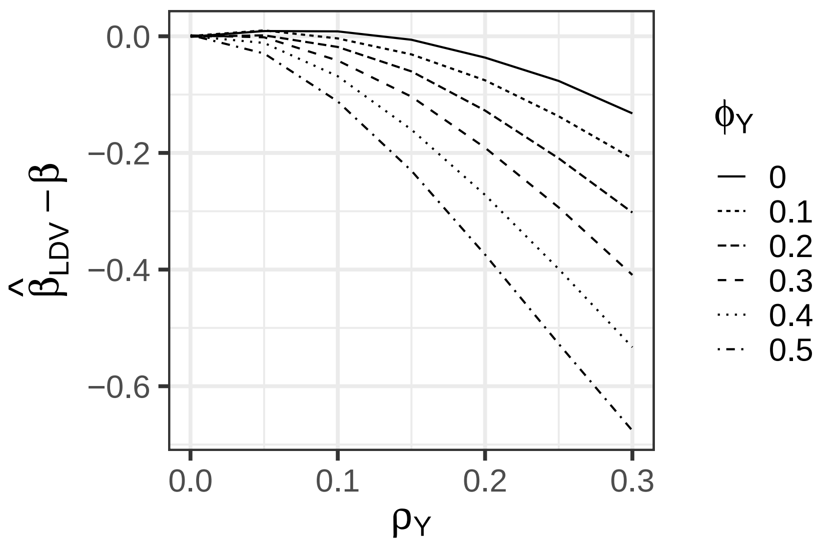


Figure 7: LDV Performance with Spatial Dependence — Bias in $\hat{\beta}$

Figures 6 & 7 present simulation results for the LDV-model estimates. Figure 6 shows that $\hat{\phi}_y$ (temporal-lag coefficient) suffers inflation bias for all ρ_y>0 (spatial-lag coefficient), with the bias magnitude increasing in both ρ_y and ϕ_y. Even when ϕ=0, substantial bias obtains—with estimates $\hat{\phi}_y$ reaching 0.18 for even the modest maximum spatial dependence considered here, ρ_y=.3—and this bias grows as ϕ_y increases, the very conditions making account of temporal dependence more important. The intuition is simple: the modeled temporal dependence can partially compensate for the missing (or, by extension, mismodeled) spatial dependence, in omitted-variable-bias fashion.

While the strength of temporal dependence is important in its own right, researchers often have greater interest in $\hat{\beta}$, for testing and estimating the 'effects' of model covariates, x. Figure 7 shows how the inflated ρ_y estimate attenuates the $\hat{\beta}$ estimates, with this induced attenuation bias also quite sizable and increasing in ρ_y and ϕ_y. This is striking given that, with ρ_y>0 and Cov$(\mathbf{x}, \mathbf{Wy})$>0, textbook discussion on omitting the spatial lag indicates *inflationary* OVB in $\hat{\beta}$. The *opposite* obtains here because that textbook inflation bias manifests so strongly in $\hat{\phi}_y$ that it induces a countervailing deflation bias in $\hat{\beta}$, demonstrating again that conventional understandings from single-dimensional analyses cannot be straightforwardly extended to TSCS contexts.[40]

[40] Beyond the bias in $\hat{\beta}$, the LDV coefficient-estimate standard errors are also consistently off (i.e., average reported s.e. overstate the standard deviation of $\hat{\beta}$), and yet, given the large biases in $\hat{\beta}$, the coverage of 95% confidence interval is zero (i.e., the estimated 95% confidence intervals *never* bound the true value in our simulations). The Appendix details results for these and other additional simulation metrics.

Furthermore, the unmodeled spatial dependence also undermines standard LM tests for serial correlation in the LDV-model estimation residuals, producing an unacceptably high false-positive rate, meaning that using 'remaining residual autocorrelation' to assess the adequacy of the LDV in addressing dependence will fail to guide specification appropriately (see Appendix).

In sum, with spatiotemporal dependence, LDV underestimates the 'impulse' effect of x_t, $\frac{\partial y_t}{\partial x_t}=\hat{\beta}$, but overestimates ϕ_y. As such, researchers may wonder how well these biases offset in long-run steady-state effect-estimates. In the LDV, the LRSS effect on unit i of permanent dx_i, is:

$$\frac{dy_{i,ss}}{dx_i} = \frac{\beta}{1-\phi_y} \text{ and } \frac{dy_{i,ss}}{dx_j} = 0 \ \forall j \neq i, \tag{26}$$

while the contemporaneous spatial steady-state effect of one-unit $d\mathbf{x}$ on $\mathbf{y}$ in the SAR model is:

$$\frac{d\mathbf{y}}{d\mathbf{x}} = (\mathbf{I} - \rho\mathbf{W})^{-1}\beta, \tag{27}$$

which is an $N \times N$ matrix of the effects, column-by-column, of dx in that column-unit on $\mathbf{y}$ (in all units). Thus, the *single* estimated LRSS 'effect of x on y' from the LDV (or any nonspatial model) is not even in the correct dimensionality of the spatial effect*s* (plural) of $d\mathbf{x}$ on $d\mathbf{y}$. Spatial dynamics imply movements in x in any unit have effects across all connected units, and movements in x in different units have different effects because units are differently connected to each other. Scalar summaries of 'Average Direct Effects (ADE)' (of x_i on y_i, inclusive of spatial dynamics) and of 'Average Indirect Effects (AIE)' (of $\mathbf{x}_{j\neq i}$ on y_i) can be obtained, respectively, by averaging the diagonal elements or by averaging off-diagonal elements of this $N \times N$ effect matrix (LeSage & Pace 2009), but even the ADE will not compare closely to the LDV's LRSS, because the LDV's temporal dynamics are quite imperfect substitutes for SAR's spatial dynamics.

The correctly spatiotemporal dynamic and LRSS effects of $d\mathbf{X}$ in the general first-order STADL, inclusive of both spatial and temporal dynamics and feedback, are given in Equations 23 and 24. Their simplifications to this STADL(sy^0, ty^1) model are:

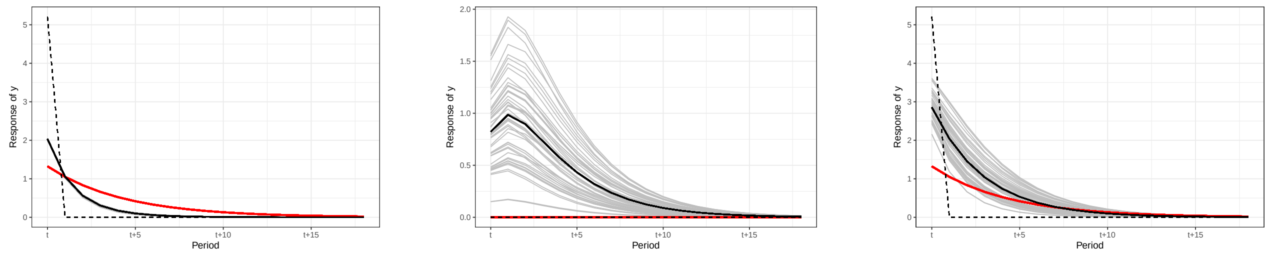
$$\text{STADL}(sy^0, ty^1) \text{ LRSS Effects: } d\mathbf{Y} = (\mathbf{I} - \phi\mathbf{I} - \rho\mathbf{W})^{-1} \cdot d\mathbf{X} \cdot \beta, \tag{28a}$$

$$\text{STADL}(sy^0, ty^1) \text{ Dynamic Effects: } d\mathbf{Y} = (\mathbf{I} - \phi\mathbf{L} - \rho\mathbf{W})^{-1} \cdot d\mathbf{X} \cdot \beta. \tag{28b}$$

For shocks to one unit, $d\mathbf{X}$ is the $N \times N$ identity matrix, $\mathbf{I}_N$, in the LRSS-effects Equation 28a, and, in the dynamic-effects Equation 28b, $d\mathbf{X}$ is that $\mathbf{I}_N$ stacked vertically T times. For shocks to all units, $d\mathbf{X}$ is an $N\times N$ block of ones. The resulting $d\mathbf{Y}$ in 28a gives the LRSS effects in all N units to shocking the column-unit, or to shocking all units. In 28b, these $N\times N$ block of effects $d\mathbf{Y}$ recurs vertically T times, period-by-period. The scalar summaries of LRSS or period-by-period ADE and AIE are found by averaging across the $N\times N$ effects block's diagonals or over all its off-diagonal elements as before. Given all this, clearly, even if the LDV model accurately recovered the LRSS average direct effect—we will show it does not—it would still produce biased estimates of these unit-specific responses.

Figures 8 & 9 illustrate all this, in one set of conditions: $\phi_y = 0.5$ and $\rho_y = 0.3$, and for one-unit shocks. Figure 8 compares the N estimated marginal period-by-period incremental response paths, i.e., impulse-response functions, a.k.a. the responses to temporary (one-period) shocks using (23) of (1) the correct STADL model: N grey, thinner response-lines, and heavier black response-line average; (2) the LDV model: *one* red, thicker response-line; and (3) the static-model: *one* dashed response-line. The 'direct' effects are of shocks to unit i on outcomes in unit i; the 'indirect' are summed responses in units $j \neq i$ to shocks in unit i; and 'total' effects sum direct and indirect. Figure 9 plots the analogous cumulative response paths to permanent (one-unit) shocks.[41] As shown analytically above, the LDV substantially underestimates the contemporaneous (same-period) effect for all N units, and it overestimates the temporal persistence, giving incorrectly slower decay. Thus, the LDV estimates *one* smaller, but more-persistent, effect, than the true STADL's *heterogeneous*, larger, quicker-decaying true effect*s*. The LDV also overestimates (underestimates) the cumulative LRSS direct (*cum* total) effect at 6.41, to which it arrives more slowly, compared to the *average* cumulative LRSS direct effect of 4.39 and total effect of 10.0 from the correct STADL, to which it arrives more quickly. (The static model, meanwhile, radically overstates direct (and total) contemporaneous effect, and badly mischaracterizes (and understates) the direct (and total) cumulative effects.) In sum, even on average—i.e., disregarding the unit-specific variation—the LDV model performs poorly (and the static nonspatial model very poorly).

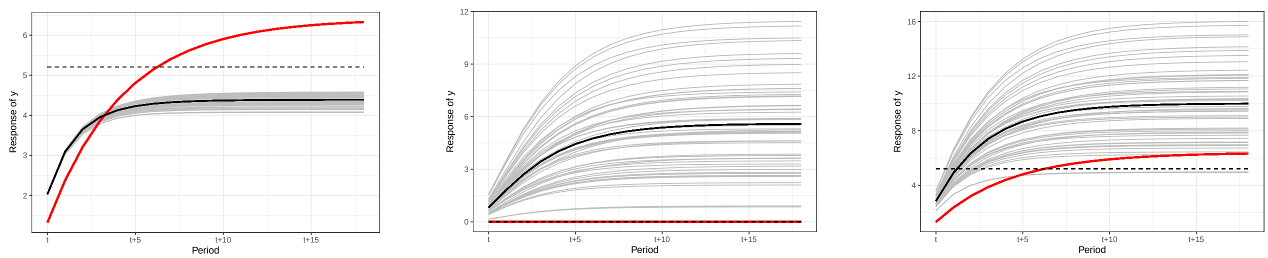
[41] The responses to all-unit shocks differ only for spatially cognizant models, and follow the same patterns as seen in Figures 8 & 9, at roughly N times greater scale.



(a) Direct **(b)** Indirect **(c)** Total

Figure 8: Response-Path Estimates of LDV Model with Spatial Dependence

Note: dotted line = static, red line = LDV, grey lines = STADL unit-by-unit, black line = STADL average units



(a) Direct **(b)** Indirect **(c)** Total

Figure 9: Cum. Response-Path Estimates of LDV Model with Spatial Dependence

Note: dotted line = static, red line = LDV, grey lines = STADL unit-by-unit, black line = STADL average units

The analogous explorations of SAR-model estimates show (Figure 10) the expected inflation bias in $\hat{\rho}$ when temporal dependence is present but unmodeled. When ϕ_y=0.05, this bias is greater than 2 times(!) the true value of ρ. As researchers more-commonly attach theoretic importance to their spatial-dependence specifications than to temporal dependence—selecting connectivity matrices to test competing theories of diffusion, e.g.—this, in itself, is more substantively meaningful than in the LDV case. Researchers interested in evaluating spatial theories in political science must attend equally highly carefully to accurately modeling temporal dynamics. Even when most aspects of one's spatial model are accurately specified (e.g., correct $\mathbf{W}$ and spatial process), failing to adequately address temporal dependence can produce wildly inaccurate understandings of the spatial processes in one's data.[42]

Regarding $\hat{\beta}$, we again observe the expected inflationary bias from the failure to model temporal dependence; however, this bias does not increase with the level of ρ_y. Why is this? First,

[42] The Appendix also shows average reported standard errors exceed the true standard deviation of the coefficient across trials and yet 95% confidence intervals rarely contain the true value (coverage probabilities well below the expected 0.95 whenever $\rho_y \neq 0$ and $\phi_y \neq 0$) because of the coefficient-estimate bias.

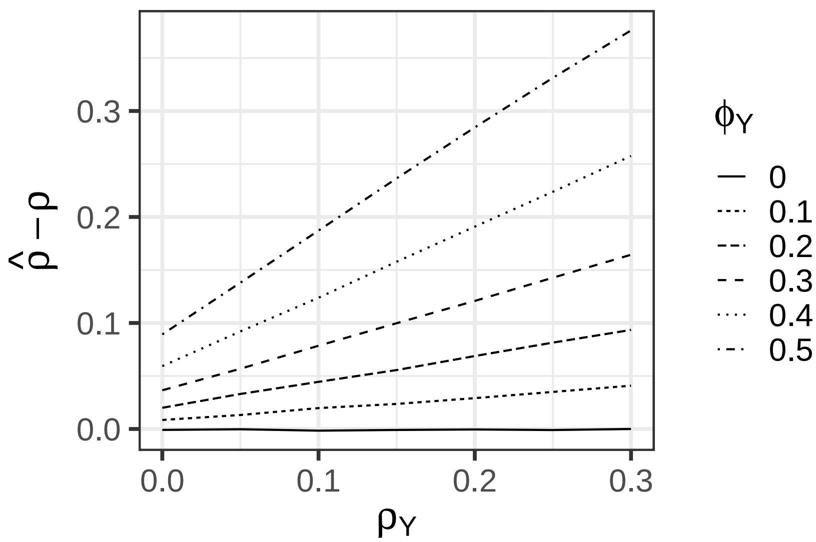


Figure 10: SAR Performance with Temporal Dependence — Bias in ρ_y

temporal dependence is often, as in our simulation, far more substantial than spatial. As such, the inflationary bias in $\hat{\beta}$ from the unmodeled temporal dynamics weighs more heavily against the downward bias from overestimated $\hat{\rho}_y$ than in the reverse scenario. Second, our simulation parameters, paralleling typical real data, set the dynamics in x also to have larger temporal than spatial dependence: ϕ_x=0.6 vs ρ_x=0.3. Thus, the correlation between $x_{i,t}$ and $y_{i,t-1}$, and so the bias from omitting the latter, is stronger than that induced by the correlation of $x_{i,t}$ and $y_{j,t}$.[43]

Although the $\hat{\beta}$ estimate (seen in Figure 11) is biased in proportion solely to the temporal-dependence misspecification, that bias plus the inflation bias in $\hat{\rho}_y$ compromises the effects estimates very notably. Recall that in spatial-autoregressive models, as in all models beyond the purely linear-additive and separable, the effect (singular) of x on y is not β, which is merely the pre-spatial impulse, but instead the effects (plural) are given by Equations 23 and 24. For scalar summaries of these multidimensional effects, one can average the diagonal or off-diagonal elements for 'average direct' and 'average indirect' effects, respectively, as previously described. Comparing the values estimated by the incorrect SAR to those from the correct STADL, we find that SAR overestimates the (one-unit shocks) average direct effect (SAR ADE=3.96 vs. STADL ADE=2.03) and radically overestimates the average total (and so even more so the average in-

[43]Verifying this, reversing the strengths of the dependencies in x to ϕ_x=0.3 and ρ_x=0.6, the relative magnitude of the bias in $\hat{\beta}$ is reduced, and the extent of the bias is affected more acutely by the level of ρ.

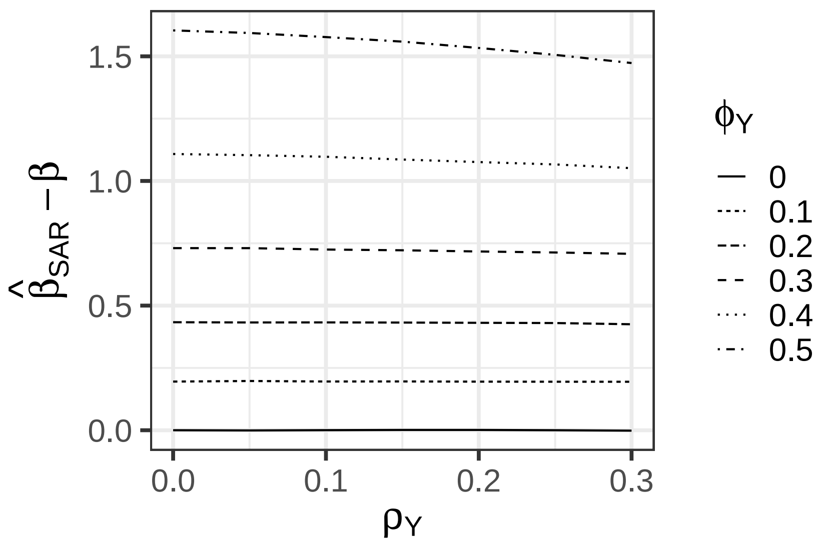


Figure 11: SAR Performance with Temporal Dependence — Bias in β

direct) contemporaneous effects (SAR ATE=10.72 vs STADL ATE=2.85). Furthermore, despite (or perhaps given) the absence of temporal dynamics from the model, the long-run, steady-state (LRSS) effects are also underestimated: (SAR LRSS ATE=10.72 vs. STADL LRSS ATE=10).

5 Empirical Reanalyses

To demonstrate the importance of these modeling choices for actual applied TSCS data-analysis, we conduct two brief reanalyses of Acemoglu et al. (2008) and of Lührmann et al. (n.d.) using our new R package, `tscsdep`.[44] Acemoglu et al. (2008) provide one of the more prominent recent empirical evaluations of the development–democracy connection, our running illustration heretofore. Particularly useful for our purposes, Acemoglu et al. (2008) account for temporal autoregressive dependence and included fixed unit & period effects, but otherwise neglect spatial dependence. In their forthcoming *APSR* article, Lührmann et al. (n.d.) develop several new country-year indices of vertical, horizontal, and diagonal political accountability, plus an overall accountability index. Much of their article is devoted to demonstrating the content, convergent and construct validity of these measures. They account for spatial dependence and include fixed

[44]Parallel Stata code forthcoming.

unit and period effects in their analyses, but omit (autoregressive) temporal dependence.

5.1 Reanalysis of Acemoglu et al. (2008) on Development & Democracy

The main finding in Acemoglu et al. (2008) is that the otherwise robust positive effect of economic development on democratization disappears when one includes fixed country effects in the model. In Table 1, we use their data to estimate four regressions that contain various combinations of fixed effects and autoregressive lags. These results starkly highlight the ways these specification choices affect one's analysis.[45]

Table 1: Reanalysis of Development & Democracy in Acemoglu et al. (2008)

	Dependent variable: Democracy (Polity IV)			
	(1)	(2)	(3)	(4)
Lagged RGDP Per Capita	0.237***	0.228***	−0.011	0.053***
	(0.01)	(0.01)	(0.027)	(.0.008)
Temporal Lag				0.746***
				(0.021)
Spatial Lag	0.138**	0.167***	0.040	0.091**
	(0.06)	(0.058)	(0.050)	(0.042)
Observations	854	854	854	854
Fixed Country Effects	No	No	Yes	No
Fixed Year Effects	No	Yes	Yes	Yes
LL	-162.34	-129.62	253.20	247.17
DoF (Parameters)	850 (4)	842 (12)	709 (145)	841 (13)
BIC	351.7	340.2	472.3	-406.6

Note: $^{*}p < 0.1;^{**} p < 0.05;^{***} p < 0.01$

In column (1), the regression includes lagged log-real-GDP-per-capita and a spatial lag created with a row-standardized nearest-neighbor weights matrix (auto-generated by `tscsdep`). Since spatial autoregression is the only spatiotemporal dependence in this model, the positive & significant coefficient on the spatial lag is unsurprising. We add fixed year effects in column (2). Since democracy trends globally over the sample period, this addition greatly improves model fit. The log-likelihood increases over 30% (-162.34 to -129.62) and the BIC decreases from 351.7 to 340.2.

[45] The replication is of their main two-way fixed-effects regression (Table 3, column 2) that uses Polity IV democracy as outcome variable.

The coefficient estimates are affected only slightly, with $\hat{\rho}$ becoming larger and more-significant.

Column (3) adds country fixed-effects. These results echo the main point in Acemoglu et al. (2008): the statistical significance of RGDP per capita on democracy disappears when we add country fixed-effects. The spatial-lag coefficient also becomes insignificant, with the country fixed-effects apparently accounting sizable time-invariant spatial clustering in both RGDP per capita and democracy. The impact on model fit, however, is less clear: LL improves greatly, but the BIC fit statistic, which penalizes for over-parameterizing the model and over-fitting the sample, also gets much worse, increasing almost 40% (340.2 to 472.3). Scholars can reasonably disagree about the model-selection implications of these comparisons; our purpose is merely to illuminate them.

Column (4) presents results from the model with by far the best BIC (-406.6) and LL close to model (3) despite 132 fewer estimated parameters. Model (4) includes both temporal & spatial lag, and period fixed-effects. The coefficients on RGDP per capita, the temporal lag, and the spatial lag are all statistically significant. As our analyses above would suggest: $\hat{\rho}$ decreases relative to models (1) and (2), as some of the dependence is temporal rather than spatial, and $\hat{\beta}$ decreases, due to the (properly) larger spatial-temporal multiplier implied by $\hat{\rho}$ and $\hat{\phi}$, which (properly) distributes the (better) estimate of this development→democracy effect across space over time.

5.2 Reanalysis of Lührmann et al. (n.d.) on Accountability & Infant Mortality

In their forthcoming *APSR* article, Lührmann et al. (n.d.) demonstrate construct validity for their overall index of political accountability by showing that it correlates (negatively) with infant mortality rates. They estimate four time-series-cross-sectional regressions, both in isolation and in combination with alternative measures of accountability taken from the World Bank and Freedom House. We conduct a brief reanalysis of their primary regression: MODEL 1 in Figure 8. The model includes the new overall accountability index and a full set of controls, including country and year fixed-effects as some account of spatial and temporal dependence, plus a regional average infant mortality variable. This regional average variable is actually a kind of spatial lag, being the average dependent-variable among regional neighbors, but it is treated as an exogenous regressor. Beyond the time-period indicators, temporal dependence and dynamics are not modeled.

The country fixed-effects account fixed (long-run) additive spatial clustering in the outcome, infant mortality rates. *Fixed* here means constant over the entire sample period (1960-2010). *Additive* means the clustering manifests as a single mean-shift, as opposed to a multiplicative effect on some observed or unobserved covariate or an autoregressive spatial dynamic process. The regional-average variable, which proxies a spatial autoregressive process, accounts for potential time-varying (long-run) spatial clustering. If there are multiple regional equilibria over time (e.g., Southeast Asia 1961-1980; Southeast Asia 1981-2000; Southeast Asia 2001-2010), though, the regional-average spatial-lag cannot account for this. Country fixed-effects cannot either.

The year fixed-effects can account 'short-run' (unique year-by-year) common shocks that are global in scope. Again, these are additive: some mean-shift each year that is common, or on-average, across all countries. The same infant-mortality shock, equal to that year's single time-dummy coefficient, hits every country. Year fixed-effects cannot account for common shocks that are regional or otherwise sub-global in nature: e.g., an infant mortality shock specific to Southeast Asia. If the relevant regions or groups of countries were known pre-analysis, regional-period shock indicators (e.g., Southeast Asia 1987) could be included in regression models, but the relevant spatio-temporal units are rarely known, and this strategy quickly overloads degrees of freedom.

An alternative strategy to account for regional common shocks is to add spatial lags in first differences to regression models. Because spatial lags represent autoregression in space—countries influence first, second and third (etc.) order neighbors with geometrically decaying impact—they provide a certain flexibility with respect to identifying the geographical boundaries of shocks that regional indicators do not. The spatial-weights matrix could connect 'k-nearest neighbors', e.g., around each country (automatically generated using `tscsdep`), whereas 'regions' must be pre-identified. Additionally, spatial lags are generally far more parsimonious than regional-period shock indicators because a single spatial-lag defines a 'neighborhood' for every sample-unit.

More generally, in STADL models, right-hand-side variables that are differenced produce short-run shocks to left-hand-side outcomes, whereas variables in levels produce long-run effects through temporal multipliers. Lührmann et al.'s MODEL 1 includes a *de facto* endogenous spatial lag in the regional averages, which are incorrectly treated as exogenous, and that we will assume are roughly specified relative to the true spatial-dependence process. Their model also includes country and year fixed effects, but no temporal dynamics, a stark omission given that infant-mortality rates are likely highly persistent temporally. We also think that regional shocks in infant mortality rates are highly plausible. Therefore, we include a temporal lag and a nearest-neighbor spatial lag in first differences in our reanalysis model:

$$y_{it} = \mathbf{x_{it}}\boldsymbol{\beta} + \phi y_{it-1} + \rho \mathbf{w_i} \Delta \mathbf{y_t} + f_i + g_t + \varepsilon_{it}, \tag{29}$$

with y_{it} being infant mortality in unit i in year t, $\mathbf{x}_{it}$ a $1{\times}k$ vector of exogenous covariates for unit-year it, $\boldsymbol{\beta}$ a $k{\times}1$ vector of coefficients, ρ the spatial-lag coefficient, $\mathbf{w_i}$ a unit-specific vector of spatial weights, $\Delta\mathbf{y}_t$ a time-t vector of differenced outcomes, f_i a fixed unit-effect, g_t a fixed period-effect, and ε_{it} an *i.i.d.* disturbance for unit-time it. Some algebraic manipulation rewrites this with a differenced outcome (which is more-convenient for expressing the likelihood):

$$\Delta y_{it} = \mathbf{x_{it}}\boldsymbol{\beta} + (\phi - 1) y_{it-1} + \rho \mathbf{w_i} \Delta \mathbf{y_t} + f_i + g_t + \varepsilon_{it}. \tag{30}$$

As the original analysis treats the regional-average variable as an exogenous regressor among $\mathbf{x}_{it}$, we retain this specification for better comparability. While this regional-average variable accounts for some spatial dependence, it is likely overestimated because temporal dependence (which is very high in infant mortality) is omitted, beyond the year-effects—which year-effects, due to regional concentration in infant-mortality shocks, likely miss considerable spatiotemporal dependence as well. Our analyses above suggest that the unfortunate consequence of this mis-estimation of the spatiotemporal dependence is that Lührmann et al. may well have *underestimated* the strength of the relationship of their political-accountability measure to infant mortality.

We replicate original results in Table 2 column one. Then, with `tscsdep`, we create a nearest-neighbor spatial weights matrix and estimate the spatiotemporal-autoregressive (STADL(sy^0, ty^1) model incorporating spatially and temporally lagged dependent-variable regressors, reported in

Table 2: Reanalysis of the Accountability / Infant Mortality Regression in Lührmann et al. (n.d.)

	Dependent variable: Infant Mortality		
	Level	Difference	Long Run
Accountability	-4.256^{***}	-0.197^{***}	-9.748
	(0.351)	(0.038)	
Foreign aid	-0.048	0.016^{***}	0.771
	(0.031)	(0.003)	
GDP/capita (ln)	-9.559^{***}	0.763^{***}	37.74
	(0.761)	(0.084)	
Economic Growth	0.033	-0.019^{***}	-0.972
	(0.024)	(0.003)	
Resource dependence	0.040^{*}	0.013^{***}	0.661
	(0.022)	(0.002)	
Economic inequality	-0.062^{**}	0.006	0.293
	(0.031)	(0.003)	
Population (ln)	-13.879^{***}	0.632^{***}	31.23
	(1.485)	(0.163)	
Urbanization	-0.142^{***}	0.023^{***}	1.135
	(0.028)	(0.003)	
Political violence	0.358^{***}	-0.016	-0.810
	(0.129)	(0.014)	
Communist	0.956	-0.784^{***}	-38.77
	(1.620)	(0.174)	
Infant mortality, regional average	0.674^{***}	0.008^{***}	0.419
	(0.020)	(0.002)	
Political corruption index	-2.907^{*}	-0.293	-14.505
	(1.905)	(0.205)	
Temporal Lag (Level)		-0.020^{***}	
		(0.002)	
Spatial Lag (Difference)		0.037^{*}	
		(0.020)	
Observations	4,354	4,312	
Fixed Country Effects	Yes	Yes	
Fixed Year Effects	Yes	Yes	

Note: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

the second column. The LRSS effects[46] of each covariate x in $\mathbf{x}_{it}$ are given in the third column. Comparing the implicit spatial steady-state implied by the regional-average variable in the original

[46] These LRSS use only the temporal multiplier as Lührmann et al. do not interpret their implicit spatial-lag regional-average as such and our added spatial lag is in changes, not levels.

regression, which ignores temporal (autoregressive) dynamics, with our estimate of the spatial steady-state effect, we estimate that the former overstates the extent of spatial dependence by nearly 38% in this comparison. More simply and starkly, comparing the first and third columns, we estimate that the spatiotemporal LRSS effect of Lührmann et al.'s political accountability on infant mortality rates (-9.748) is more than double the 'effect' they reported ($\hat{\beta}=-4.256$), which mostly ignores these important spatial and temporal dynamic dependencies.

6 Conclusion

This paper considers the implications of the multidimensional dependence, the dynamics in both space and time, typically manifest in TSCS data for the currently common practice in empirical analyses to privilege one of temporal or spatial dependence to the complete or relative neglect of the other. With dependence in both space and time, however, modeling dependence in one dimension while neglecting the other results in biases that differ from those considered heretofore in textbook treatments of temporal and spatial dependence. We detailed and demonstrated these biases analytically and in simulations and applications. To address these issues, we proposed a spatiotemporal model, the first-order STADL, which nests many of the most-widely used space-time specifications in political science (e.g., the first-order LDV, ADL, SAR, SDM), and discussed the interpretation of the varieties of spatiotemporally dynamic effects different STADL specifications entail. We suggested that beginning with this more-general STADL specification and using Wald tests to guide model refinement reduces the risk of unmodeled dynamics, a necessary condition for valid estimation and inferences regarding parameters and effects. To better enable researchers to adopt the strategies presented here, we developed R package, `tscsdep` (see Appendix for detail; GitHub to download) to construct common weights matrices, including for unbalanced panels, estimate the STADL model, and generate STADL dynamic and LRSS effects.

To mention possible drawbacks, and work remaining to be done, with our recommended STADL approach for TSCS data analysis: we have not addressed the topic of order-specification decisions, focusing instead on source & dimension specification, and we did not raise the possibility of overfitting STADL models to sample idiosyncrasies. We believe effective approaches to these challenges extend naturally from time-series and spatial econometrics. For instance, autocorrelation and partial autocorrelation (AC, PAC) functions used to guide time-series order specification can be extended to *spatiotemporal* AC and PAC functions. Likewise, out-of-sample forecasting is the gold-standard safeguard against overfitting and is similarly extendable to spatiotemporal TSCS contexts. These projects head our research agenda going forward.

References

Acemoglu, D., Johnson, S., Robinson, J. A. & Yared, P. (2008), 'Income and democracy', *American Economic Review* **98**(3), 808–42.

Acemooglu, D., Johnson, S., Robinson, J. A. & Yared, P. (2008), 'Income and democracy', *American Economic Review* **98**(3), 808–842.

Achen, C. H. (2000), 'why lagged dependent variables can suppress the explanatory power of other independent variables'. Presented at the Annual Meeting for the Society of Political Methodology, UCLA.

Alexander, G. (2001), 'Institutions, path dependence, and democratic consolidation', *Journal of theoretical politics* **13**(3), 249–269.

Anselin, L. (1988), *Spatial econometrics: methods and models*, Vol. 4, Springer Science & Business Media.

Anselin, L. (2001), Spatial econometrics, *in* 'Handbook of Econometrics: Volume 1, Econometric'.

Anselin, L., Bera, A. K., Florax, R. & Yoon, M. J. (1996*a*), 'Simple diagnostic tests for spatial dependence', *Regional science and urban economics* **26**(1), 77–104.

Anselin, L., Bera, A. K., Florax, R. & Yoon, M. J. (1996*b*), 'Simple diagnostic tests for spatial dependence', *Regional Science and Urban Economics* **26**(1), 77–104.

Baybeck, B. & Huckfeldt, R. (2002), 'Spatially dispersed ties among interdependent citizens: connecting individuals and aggregates', *Political Analysis* pp. 261–275.

Beck, N. & Katz, J. N. (1995), 'What to do (and not to do) with time-series cross-section data', *American Political Science Review* **89**(03), 634–647.

Beck, N. & Katz, J. N. (2011), 'Modeling dynamics in time-series-cross-section political economy data', *Annual Review of Political Science* **14**, 331–352.

Betz, T., Cook, S. J. & Hollenbach, F. M. (2020), 'Bias from network misspecification under spatial dependence', *Political Analysis* p. 1–7.

Box-Steffensmeier, J. M., Freeman, J. R., Hitt, M. P. & Pevehouse, J. C. (2014), *Time series analysis for the social sciences*, Cambridge University Press.

Buhaug, H. & Gleditsch, K. S. (2008), 'Contagion or confusion? why conflicts cluster in space', *International Studies Quarterly* **52**(2), 215–233.

Cook, S. J., Hays, J. C. & Franzese, R. J. (2015), Model specification and spatial interdependence. Presented at the Annual Meeting for the Society of Political Methodology, University of Rochester.

Cook, S. J., Hays, J. C. & Franzese, R. J. (2020), Model specification and spatial interdependence, *in* L. Curini & R. Franzese, eds, 'The SAGE Handbook of Research Methods in Political Science and International Relation', SAGE, London, chapter 39, pp. 730–747.

Diamond, L. (1994), 'Rethinking civil society: Toward democratic consolidation', *Journal of democracy* **5**(3), 4–17.

Drolc, C. A., Gandrud, C. & Williams, L. K. (2019), 'Taking time (and space) seriously: How scholars falsely infer policy diffusion from model misspecification', *Policy Studies Journal* .

Elhorst, J. P. (2001), 'Dynamic models in space and time', *Geographical Analysis* **33**(2), 119–140.

Elhorst, J. P. (2014), *Spatial econometrics: from cross-sectional data to spatial panels*, Vol. 479, Springer.

Elkink, J. A. (2011), 'The international diffusion of democracy', *Comparative Political Studies* **44**(12), 1651–1674.

Fotheringham, A. S. & Wong, D. W. (1991), 'The modifiable areal unit problem in multivariate statistical analysis', *Environment and planning A* **23**(7), 1025–1044.

Franzese, R. J. & Hays, J. C. (2007), 'Spatial econometric models of cross-sectional interdependence in political science panel and time-series-cross-section data', *Political Analysis* **15**(2), 140–164.

Franzese, R. J. & Hays, J. C. (2008*a*), 'Empirical methods of spatial interdependence', *Oxford Handbook of Political Methodology* pp. 570–604.

Franzese, R. J. & Hays, J. C. (2008*b*), 'Interdependence in comparative politics: Substance, theory, empirics, substance', *Comparative Political Studies* **41**(4-5), 742–780.

Freeman, J. R. (1989), 'Systematic sampling, temporal aggregation, and the study of political relationships', *Political analysis* pp. 61–98.

Gibbons, S. & Overman, H. G. (2012), 'Mostly pointless spatial econometrics?*', *Journal of Regional Science* **52**(2), 172–191.

Gleditsch, K. S. & Ward, M. D. (2006), 'Diffusion and the international context of democratization', *International organization* **60**(04), 911–933.

Graham, E. R., Shipan, C. R. & Volden, C. (2013), 'The diffusion of policy diffusion research in political science', *British Journal of Political Science* pp. 673–701.

Halleck Vega, S. & Elhorst, J. P. (2015), 'The slx model', *Journal of Regional Science* **55**(3), 339–363.

Hays, J. C., Kachi, A. & Franzese Jr, R. J. (2010), 'A spatial model incorporating dynamic, endogenous network interdependence: A political science application', *Statistical Methodology* **7**(3), 406–428.

Hendry, D. F. (1995), *Dynamic econometrics*, Oxford University Press.

Huckfeldt, R. & Sprague, J. (1987), 'Networks in context: The social flow of political information', *The American Political Science Review* pp. 1197–1216.

Juhl, S. (2020), 'The sensitivity of spatial regression models to network misspecification', *Political Analysis* **28**(1), 1–19.

Keck, M. E., Sikkink, K. et al. (1998), *Activists beyond borders: Advocacy networks in international politics*, Cornell University Press.

Keele, L. & Kelly, N. J. (2006), 'Dynamic models for dynamic theories: The ins and outs of lagged dependent variables', *Political Analysis* **14**(2), 186–205.

Kirkland, J. H. (2011), 'The relational determinants of legislative outcomes: Strong and weak ties between legislators', *The Journal of Politics* **73**(3), 887–898.

LeSage, J. P. & Pace, R. K. (2014), 'The biggest myth in spatial econometrics', *Econometrics* **2**(4), 217–249.
URL: *https://www.mdpi.com/2225-1146/2/4/217*

LeSage, J. & Pace, R. K. (2009), *Introduction to spatial econometrics*, CRC press.

Lipset, S. M. (1959), 'Some social requisites of democracy: Economic development and political legitimacy', *The American political science review* **53**(1), 69–105.

Lührmann, A., Marquardt, K. L. & Mechkova, V. (n.d.), 'Constraining governments: New indices of vertical, horizontal, and diagonal accountability', *American Political Science Review* .

Neumayer, E. & Plümper, T. (2016), 'W', *Political Science Research and Methods* **4**(1), 175–193.

Neumayer, E. & Plümper, T. (2017), *Robustness tests for quantitative research*, Cambridge University Press.

Przeworski, A., Alvarez, R. M., Alvarez, M. E., Cheibub, J. A., Limongi, F. & Limongi Neto, F. P. (2000), *Democracy and development: Political institutions and well-being in the world, 1950-1990*, Vol. 3, Cambridge University Press.

Robinson, J. A. (2006), 'Economic development and democracy', *Annu. Rev. Polit. Sci.* **9**, 503–527.

Rüttenauer, T. (2019), 'Spatial regression models: A systematic comparison of different model specifications using monte carlo experiments', *Sociological Methods & Research* .

Simmons, B. A. & Elkins, Z. (2004), 'The globalization of liberalization: Policy diffusion in the international political economy', *American political science review* pp. 171–189.

Starr, H. (1991), 'Democratic dominoes: Diffusion approaches to the spread of democracy in the international system', *Journal of Conflict Resolution* **35**(2), 356–381.

Stimson, J. A. (1985), 'Regression in space and time: A statistical essay', *American Journal of Political Science* **29**(4), 914–947.

Stram, D. O. & Wei, W. W. (1986), 'Temporal aggregation in the arima process', *Journal of Time Series Analysis* **7**(4), 279–292.

Tam Cho, W. K. & Fowler, J. H. (2010), 'Legislative success in a small world: Social network analysis and the dynamics of congressional legislation', *The Journal of Politics* **72**(1), 124–135.

Whitten, G. D., Williams, L. K. & Wimpy, C. (2021), 'Interpretation: the final spatial frontier', *Political Science Research and Methods* **9**(1), 140–156.

Wilson, S. E. & Butler, D. M. (2007), 'A lot more to do: The sensitivity of time-series cross-section analyses to simple alternative specifications', *Political Analysis* **15**(2), 101–123.

Wimpy, C., Williams, L. K. & Whitten, G. D. (2021), 'X marks the spot: Unlocking the treasure of spatial-x models', *The Journal of Politics* **0**(ja), null.
URL: *https://doi.org/10.1086/710089*

Supplemental Appendix

STADL Up! The Spatio-Temporal Autoregressive Distributed Lag Model for TSCS Data Analysis

1 TSCS Data in the *APSR*, *AJPS* and *JOP*

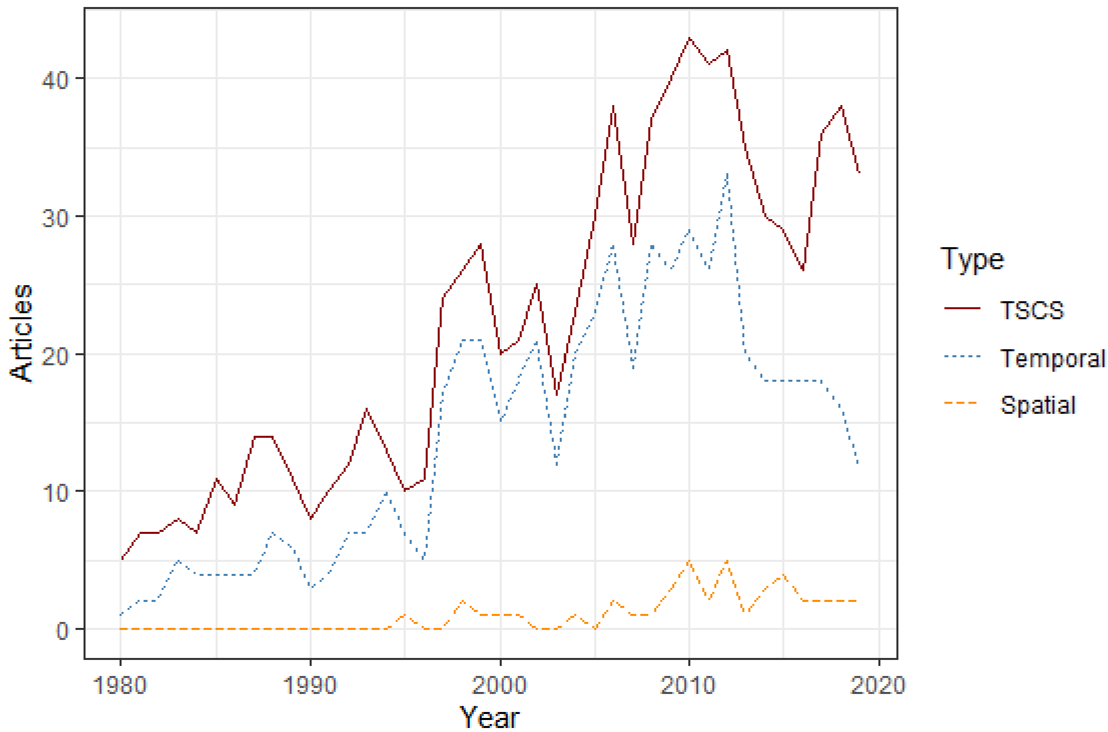
To calculate the number of articles that use time-series cross-sectional data, we analyzed 7336 research articles appearing in *The American Political Science Review*, *The American Journal of Political Science*, and *The Journal of Politics*, from 1980 to 2019. Using this set of articles, we counted the occurrence(s) of a list of keywords related to time-series cross-sectional data in each article. For the years 1980 to 2014, we use the data and metadata provided by JSTOR's API.[1] To collect the recent articles that are not covered by JSTOR's API, we directly scraped the text from each journal's website, the same keywords are used to count the number of occurrences in each article.

These results are summarized in Figure 1 which provides the yearly count of articles using keywords associated with TSCS data analysis. Specifically, TSCS (solid line) gives the number of articles using at least one of the following keywords: 'time series cross section(al)', 'tscs', 'panel data.'[2] Within the set of TSCS articles, we then count those that use keywords consistent with Temporal analysis (dotted line) – e.g., 'time series', 'time serial', 'temporal autocorrelation', 'temporal correlation','temporal dependence', 'temporal dynamics', 'time dependence', 'time lag(ged)', 'time lagged dependent', 'serial correlation', 'serially correlated', 'serial dependence.' – and Spatial analysis (dashed line) – e.g., 'spatial dependence', 'spatial interdependence', 'spatial autocorrelation', 'spatial correlation', 'spatially correlated', 'spatial lag', 'spatial-lag dependent', 'spatially lagged', 'spatially lagged dependent.'

[1]JSTOR's API has different time coverage for each journal. It provides upto 2014, 2015, 2018 for APSR, JOP, AJPS, respectively.

[2]Since JSTOR's API only provides the count of words up to trigrams, we use the keyword 'time series cross' instead of 'time series cross section' or 'time series cross sectional'.

Figure 1: Count of Articles Using TSCS Data in the Top-3, 1980-2019



The results demonstrate that TSCS data remain widely used, with 33 articles appearing in the top-3 (APSR, AJPS, and JOP) in 2019 alone. Few of these, however, seem to meaningfully consider both temporal and spatial dependence. In 2019, for example, of the 33 TSCS articles only 12 used keywords consistent with temporal analysis and only 2 with spatial analysis. As such, at most 2 could have jointly considered both temporal and spatial analysis, as our manuscript suggests in necessary.

2 Monte Carlo Analysis

2.1 Additional Design Details

The spatial locations for the units are generated by twice taking N draws from a standard uniform to create xy-coordinates for each unit. The w_{ij} relative connections between units are then generated using a k-Nearest Neighbor algorithm with $k = 5$, returning a binary N-by-N matrix $\mathbf{W_N}$ with each element $w_{ij} = 1$ for the five closest j to i and 0 for all others (and all $w_{ii} = 0$ along the diagonal). The Kronecker product of this matrix and a T-dimensional identity matrix produces $\mathbf{W}$, an NT-by-NT matrix with each N-by-N block $\mathbf{W_N}$ along the prime diagonal giving the dyadic relations, which are assumed constant over time. We also assume that this same $\mathbf{W}$ operates with respect to spatial dynamics in $\mathbf{y}$, $\mathbf{x}$, $\mathbf{u}$, and $\mathbf{e}$, and that it is known to the researcher.[3]

In order to focus on how varying the strength of ϕ_y and ρ_y affect model performance, we fix the parameters for $\beta = 2$, $\phi_x = 0.6$, and $\rho_x = 0.3$, and confine attention to variation in the temporal dependence – $\phi_y = \{0,0.1,\ldots,0.4,0.5\}$ – and spatial dependence – $\rho_y = \{0,0.05,\ldots,0.25,0.3\}$ – in $\mathbf{y}$.[4] We are primarily interested in understanding how well the LDV and SAR models recover β, however, we also examine how the estimates of ϕ_y and ρ_y are affected.

2.2 Lagrange Multiplier Tests

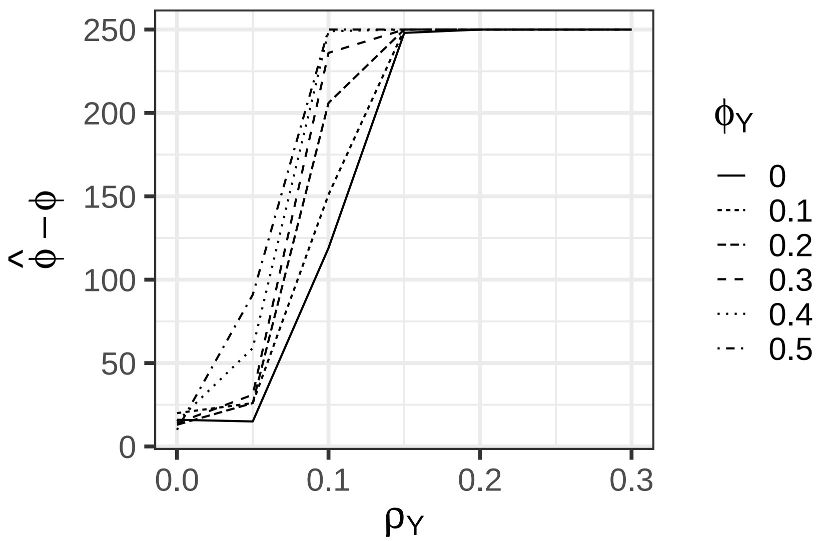
Current best practice (as advised by Beck and Katz (2011), e.g.) suggests post-estimation diagnostic tests for remaining serial dependence in estimated residuals, specifically Lagrange Multiplier tests of auxiliary regressions of estimated residuals on their lags. Unfortunately, these post-estimation tests for remaining *temporal* dependence will lead researchers astray when there is (unmodeled, i.e. remaining) residual *spatial* autocorrelation. Here, too, the unmodeled spatial

[3]Noteworthy–in that these following are also commonly issues in TSCS data-analysis–among the simplifying assumptions in our simulations are (1) no parameter heterogeneity and (2) all regressors $\mathbf{X}$ are exogenous.

[4]The extent of the bias in these parameter estimates is a function of ϕ_x and ρ_x, but for tractability we fix and do not estimate these two parameters.

dependence is "mistaken" for temporal dependence, causing researchers to over-reject the null, the frequency of which false-positive rate, intuitively, increases in ρ_y as shown in Figure 2). The LM test for residual serial correlation has power against the incorrect alternative under these circumstances, erroneously registering the spatial dependence as temporal dependence, leading researchers to take inappropriate remedial actions – e.g., modeling higher-order time-lags of the outcome – rather than addressing the truly spatial cause of the dependence in the residuals.

Figure 2: False-Positive Rate of Lagrange Multiplier test with Spatial Dependence



2.3 STADL Results

In the main text we demonstrate the bias of the SAR and the LDV models when there is unmodeled temporal or spatial dependence, respectively. We also imply that since the STADL model accounts for both temporal and spatial dependence, that it should be unbiased under these conditions. Here we demonstrate that explicitly, showing that under all conditions evaluated in our simulations the STADL model is an unbiased estimator of β (Figure 3), ρ (Figure 4), and ϕ (Figure 5).

Figure 3: STADL performance with Spatio-temporal dependence – Bias in β

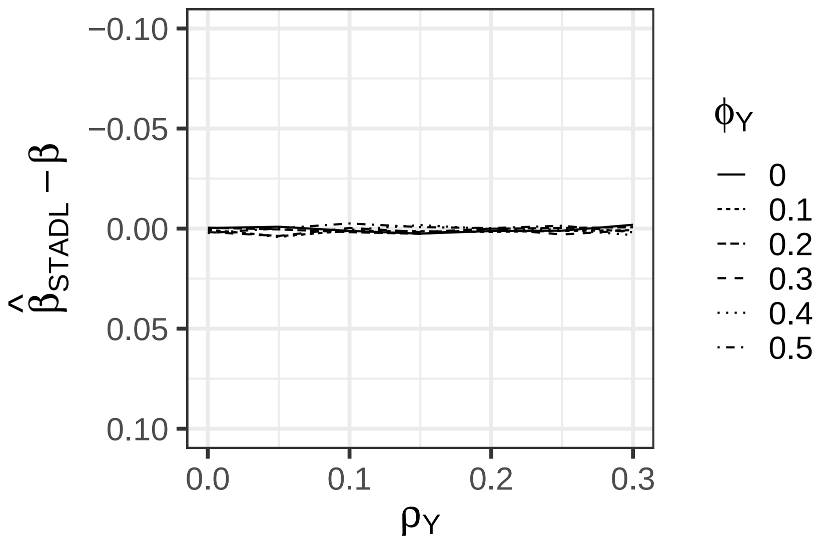


Figure 4: STADL performance with Spatio-temporal dependence – Bias in ρ

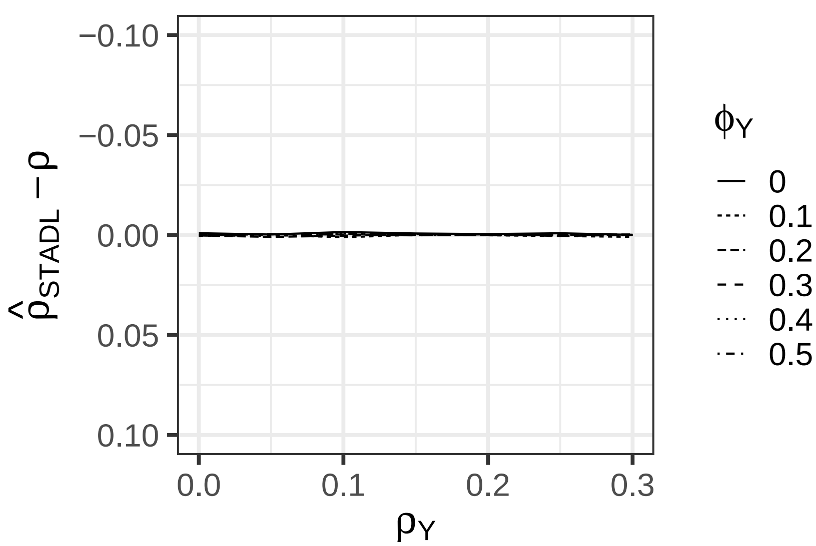
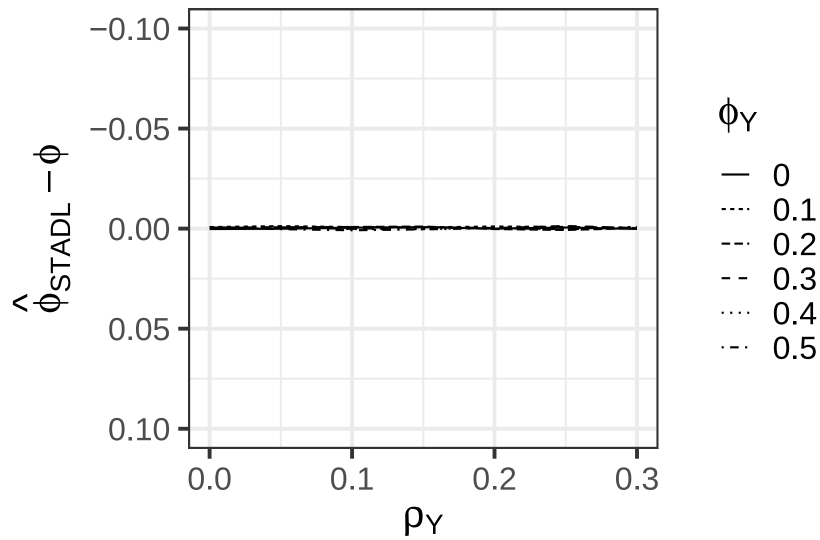


Figure 5: STADL performance with Spatio-temporal dependence – Bias in ϕ



2.4 Additional Model Comparisons

In the main text we focus exclusively on bias when evaluating the models under different simulated conditions. Here we report additional quantities of interest, including bias (Bias), average standard error (Avg. SE), the standard deviation of the empirical distribution (SD), mean square error (MSE), and the coverage probabilities (CP).[5] Since reporting all of the simulated conditions in this manner would be unwieldy, we focus on 4 values of ρ (0.0, 0.1, 0.2, 0.3), within each table, and 6 values for ϕ (0.0, ..., 0.5). Each Table (1-6) reports the results for a set value of ϕ (ex. Table 1 is $\phi = 0$), and different values of ρ (with values increasing as one moves down the Table). The results are consistent with what we would expect, in that failing to properly account for dependence between the observations induces bias, inaccurate standard errors (as demonstrated by the deviation of the Avg. SE and the SD), increased mean square error, and confidence intervals that rarely bound the true value (as indicated by CP).

[5] The coverage probabilities (CP) are calculated using the 95% confidence intervals (CIs) for each models sample coefficient and standard error estimate. The reported value for CP indicates the proportion of trials for which the 95% CI contains the true coefficient.

Table 1: Simulation Results, $\phi = 0.0$

		β				ϕ		ρ	
		Static	LDV	SAR	STADL	LDV	STADL	SAR	STADL
$\rho = 0.0$	Bias	-0.001	-0.001	0.000	-0.000	0.000	0.000	-0.001	-0.001
	Avg. SE	0.011	0.027	0.012	0.027	0.013	0.013	0.010	0.010
	SD	0.010	0.025	0.011	0.026	0.013	0.013	0.010	0.010
	MSE	0.000	0.001	0.000	0.001	0.000	0.000	0.000	0.000
	CP	0.956	0.976	0.956	0.968	0.964	0.964	0.960	0.964
$\rho = 0.1$	Bias	0.065	0.008	0.000	0.001	0.027	-0.000	-0.002	-0.002
	Avg. SE	0.011	0.028	0.012	0.026	0.013	0.013	0.010	0.010
	SD	0.010	0.029	0.012	0.028	0.014	0.014	0.010	0.010
	MSE	0.004	0.001	0.000	0.001	0.001	0.000	0.000	0.000
	CP	0.000	0.936	0.956	0.944	0.456	0.944	0.952	0.956
$\rho = 0.2$	Bias	0.150	-0.037	0.001	0.001	0.088	0.000	-0.000	-0.000
	Avg. SE	0.013	0.031	0.012	0.026	0.014	0.013	0.009	0.009
	SD	0.012	0.031	0.013	0.027	0.015	0.013	0.009	0.010
	MSE	0.023	0.002	0.000	0.001	0.008	0.000	0.000	0.000
	CP	0.000	0.808	0.936	0.956	0.000	0.940	0.948	0.952
$\rho = 0.3$	Bias	0.251	-0.132	-0.002	-0.002	0.176	0.000	0.000	-0.000
	Avg. SE	0.017	0.036	0.013	0.025	0.016	0.012	0.008	0.009
	SD	0.012	0.033	0.013	0.026	0.016	0.012	0.009	0.010
	MSE	0.063	0.019	0.000	0.001	0.031	0.000	0.000	0.000
	CP	0.000	0.032	0.960	0.936	0.000	0.944	0.932	0.940

Table 2: Simulation Results, $\phi = 0.1$

		β				ϕ		ρ	
		Static	LDV	SAR	STADL	LDV	STADL	SAR	STADL
$\rho = 0.0$	Bias	0.200	0.000	0.195	0.001	-0.001	-0.001	0.009	0.000
	Avg. SE	0.011	0.027	0.013	0.027	0.012	0.012	0.009	0.009
	SD	0.011	0.026	0.013	0.026	0.012	0.012	0.011	0.010
	MSE	0.040	0.001	0.038	0.001	0.000	0.000	0.000	0.000
	CP	0.000	0.960	0.000	0.952	0.956	0.960	0.804	0.936
$\rho = 0.1$	Bias	0.283	-0.004	0.195	0.001	0.034	-0.001	0.020	0.001
	Avg. SE	0.012	0.028	0.013	0.027	0.012	0.012	0.009	0.009
	SD	0.013	0.028	0.014	0.026	0.012	0.011	0.009	0.008
	MSE	0.080	0.001	0.038	0.001	0.001	0.000	0.000	0.000
	CP	00.000	0.968	0.000	0.964	0.248	0.968	0.416	0.968
$\rho = 0.2$	Bias	0.389	-0.075	0.195	0.002	0.105	-0.001	0.029	0.000
	Avg. SE	0.015	0.032	0.013	0.026	0.013	0.012	0.008	0.009
	SD	0.012	0.029	0.013	0.026	0.012	0.011	0.009	0.009
	MSE	0.151	0.007	0.038	0.001	0.011	0.000	0.001	0.000
	CP	0.000	0.352	0.000	0.948	0.000	0.956	0.080	0.956
$\rho = 0.3$	Bias	0.529	-0.210	0.195	0.001	0.207	-0.000	0.041	0.001
	Avg. SE	0.020	0.037	0.013	0.025	0.014	0.011	0.008	0.009
	SD	0.014	0.030	0.015	0.025	0.012	0.011	0.008	0.009
	MSE	0.280	0.045	0.038	0.001	0.043	0.000	0.002	0.000
	CP	0.000	0.000	0.000	0.944	0.000	0.960	0.000	0.932

Table 3: Simulation Results, $\phi = 0.2$

		β				ϕ		ρ	
		Static	LDV	SAR	STADL	LDV	STADL	SAR	STADL
$\rho = 0.0$	Bias	0.447	-0.001	0.433	-0.000	-0.000	-0.000	0.020	0.000
	Avg. SE	0.012	0.027	0.014	0.027	0.011	0.011	0.010	0.008
	SD	0.013	0.027	0.015	0.027	0.011	0.011	0.010	0.008
	MSE	0.200	0.001	0.188	0.001	0.000	0.000	0.001	0.000
	CP	0.000	0.956	0.000	0.952	0.940	0.940	0.432	0.948
$\rho = 0.1$	Bias	0.553	-0.018	0.433	0.002	0.041	-0.001	0.044	0.000
	Avg. SE	0.014	0.029	0.014	0.027	0.011	0.011	0.009	0.008
	SD	0.014	0.027	0.015	0.026	0.011	0.011	0.010	0.009
	MSE	0.306	0.001	0.187	0.001	0.002	0.000	0.002	0.000
	CP	0.000	0.912	0.000	0.956	0.040	0.944	0.000	0.924
$\rho = 0.2$	Bias	0.694	-0.127	0.431	0.000	0.125	0.000	0.069	0.000
	Avg. SE	0.019	0.033	0.015	0.026	0.012	0.010	0.008	0.008
	SD	0.015	0.029	0.015	0.026	0.012	0.011	0.009	0.008
	MSE	0.482	0.017	0.186	0.001	0.016	0.000	0.005	0.000
	CP	0.000	0.016	0.000	0.940	0.000	0.944	0.000	0.948
$\rho = 0.3$	Bias	0.882	-0.302	0.425	-0.001	0.235	-0.000	0.093	-0.000
	Avg. SE	0.026	0.038	0.015	0.025	0.013	0.010	0.007	0.008
	SD	0.016	0.030	0.016	0.026	0.011	0.010	0.007	0.008
	MSE	0.779	0.092	0.181	0.001	0.055	0.000	0.009	0.000
	CP	0.000	0.000	0.000	0.948	0.000	0.952	0.000	0.960

Table 4: Simulation Results, $\phi = 0.3$

		β				ϕ		ρ	
		Static	LDV	SAR	STADL	LDV	STADL	SAR	STADL
$\rho = 0.0$	Bias	0.758	0.001	0.731	0.002	-0.000	-0.001	0.037	0.000
	Avg. SE	0.015	0.027	0.017	0.027	0.010	0.010	0.010	0.007
	SD	0.014	0.027	0.017	0.027	0.010	0.010	0.010	0.007
	MSE	0.575	0.001	0.534	0.001	0.000	0.000	0.001	0.000
	CP	0.000	0.948	0.000	0.940	0.944	0.932	0.056	0.940
$\rho = 0.1$	Bias	0.898	-0.042	0.725	-0.000	0.049	0.000	0.079	-0.000
	Avg. SE	0.018	0.029	0.018	0.026	0.010	0.010	0.009	0.007
	SD	0.016	0.029	0.018	0.028	0.010	0.010	0.010	0.007
	MSE	0.806	0.003	0.526	0.001	0.003	0.000	0.006	0.000
	CP	0.000	0.708	0.000	0.960	0.008	0.944	0.000	0.932
$\rho = 0.2$	Bias	1.090	-0.191	0.717	0.000	0.143	-0.000	0.121	-0.000
	Avg. SE	0.025	0.033	0.018	0.025	0.011	0.009	0.009	0.007
	SD	0.017	0.027	0.017	0.025	0.010	0.009	0.007	0.006
	MSE	1.189	0.037	0.515	0.001	0.021	0.000	0.015	0.000
	CP	0.000	0.000	0.000	0.952	0.000	0.944	0.000	0.964
$\rho = 0.3$	Bias	1.370	-0.409	0.708	0.001	0.260	-0.000	0.164	-0.000
	Avg. SE	0.037	0.038	0.019	0.024	0.011	0.009	0.008	0.007
	SD	0.018	0.028	0.018	0.026	0.009	0.010	0.007	0.007
	MSE	1.877	0.168	0.501	0.001	0.068	0.000	0.027	0.000
	CP	0.000	0.000	0.000	0.924	0.000	0.936	0.000	0.948

Table 5: Simulation Results, $\phi = 0.4$

		β				ϕ		ρ	
		Static	LDV	SAR	STADL	LDV	STADL	SAR	STADL
$\rho = 0.0$	Bias	1.161	0.000	1.108	0.000	0.000	0.000	0.059	0.000
	Avg. SE	0.020	0.027	0.022	0.027	0.008	0.009	0.011	0.006
	SD	0.016	0.028	0.020	0.028	0.009	0.009	0.010	0.007
	MSE	1.347	0.001	1.228	0.001	0.000	0.000	0.004	0.000
	CP	0.000	0.940	0.000	0.940	0.940	0.936	0.000	0.948
$\rho = 0.1$	Bias	1.354	-0.069	1.097	0.001	0.057	0.000	0.124	-0.001
	Avg. SE	0.025	0.029	0.023	0.026	0.008	0.008	0.010	0.006
	SD	0.020	0.027	0.021	0.027	0.008	0.008	0.008	0.006
	MSE	1.833	0.005	1.204	0.001	0.003	0.000	0.015	0.000
	CP	0.000	0.336	0.000	0.940	0.000	0.956	0.000	0.960
$\rho = 0.2$	Bias	1.635	-0.272	1.076	0.000	0.161	-0.000	0.191	-0.000
	Avg. SE	0.036	0.034	0.024	0.025	0.009	0.008	0.009	0.006
	SD	0.020	0.027	0.021	0.025	0.008	0.008	0.008	0.006
	MSE	2.673	0.075	1.158	0.001	0.026	0.000	0.036	0.000
	CP	0.000	0.000	0.000	0.936	0.000	0.932	0.000	0.944
$\rho = 0.3$	Bias	2.079	-0.533	1.051	0.003	0.280	-0.001	0.258	0.000
	Avg. SE	0.056	0.038	0.026	0.023	0.009	0.007	0.008	0.006
	SD	0.022	0.023	0.021	0.023	0.006	0.007	0.006	0.006
	MSE	4.324	0.285	1.106	0.001	0.079	0.000	0.066	0.000
	CP	0.000	0.000	0.000	0.928	0.000	0.956	0.000	0.968

Table 6: Simulation Results, $\phi = 0.5$

		β				ϕ		ρ	
		Static	LDV	SAR	STADL	LDV	STADL	SAR	STADL
$\rho = 0.0$	Bias	1.698	0.002	1.604	0.002	-0.001	-0.001	0.089	-0.000
	Avg. SE	0.027	0.026	0.031	0.026	0.007	0.007	0.013	0.005
	SD	0.020	0.027	0.024	0.027	0.007	0.007	0.010	0.005
	MSE	2.884	0.001	2.573	0.001	0.000	0.000	0.008	0.000
	CP	0.000	0.936	0.000	0.936	0.944	0.948	0.000	0.948
$\rho = 0.1$	Bias	1.982	-0.112	1.577	-0.003	0.067	0.001	0.187	-0.001
	Avg. SE	0.036	0.028	0.032	0.025	0.007	0.007	0.012	0.005
	SD	0.020	0.027	0.024	0.025	0.007	0.007	0.009	0.006
	MSE	3.931	0.013	2.488	0.001	0.005	0.000	0.035	0.000
	CP	0.000	0.020	0.000	0.960	0.000	0.940	0.000	0.916
$\rho = 0.2$	Bias	2.428	-0.374	1.534	-0.000	0.177	-0.000	0.284	0.000
	Avg. SE	0.055	0.034	0.034	0.024	0.007	0.007	0.010	0.005
	SD	0.022	0.022	0.024	0.022	0.005	0.006	0.007	0.005
	MSE	5.897	0.141	2.352	0.000	0.031	0.000	0.081	0.000
	CP	0.000	0.000	0.000	0.964	0.000	0.976	0.000	0.948
$\rho = 0.3$	Bias	3.207	-0.676	1.473	0.002	0.293	-0.000	0.376	0.000
	Avg. SE	0.095	0.037	0.036	0.022	0.006	0.006	0.008	0.005
	SD	0.027	0.019	0.025	0.022	0.004	0.006	0.004	0.006
	MSE	10.285	0.458	2.170	0.000	0.086	0.000	0.141	0.000
	CP	0.000	0.000	0.000	0.952	0.000	0.944	0.000	0.940

2.5 Spatiotemporal Error Autocorrelation

In the main text we focus primarily on model performance under different levels of spatial interdependence (i.e., ρ) and serial autodependence (i.e., ϕ) in the outcome directly, reflecting the wide use of the SAR and LDV model in applied work. However, we might also be interested in how these models perform under varying levels of spatial and temporal error autocorrelation (λ and δ respectively). To evaluate this, we generate data from a STADL(se^0, te^1):

$$\mathbf{y}_t = \mathbf{x}_t\beta + (\mathbf{I} - \delta\mathbf{L} - \lambda\mathbf{W})^{-1}\boldsymbol{\varepsilon}_{\mathbf{y}},$$

with λ and δ determining the level of spatial and temporal error autocorrelation.[6] Below we report various quantities of interest (bias, average standard error, standard deviation, mean square error, and the coverage probabilities) for a subset of the simulated conditions: 4 values of λ (0.0, 0.1, 0.2, 0.3), within each table, and 6 values for δ (0.0, ..., 0.5). Each Table (7-12) reports the results for a set value of δ (ex. Table 7 is $\delta = 0$), and different values of λ (with values increasing as one moves down the Table). We focus on the estimator of β, since it is common across models and reflects the total effect of $\mathbf{x}_t$ on $\mathbf{y}_t$ under our simulated conditions.

The results are consistent with what we would expect, so we only briefly describe them globally here. First, under either (or both) forms of error dependence, the static model is unbiased by produces overly confident standard errors (as seen by the difference between Avg. SE and SD), resulting in poor coverage. Second, the respective error correlation models (SCE and SEM) are also always unbiased, however, they each produce overly confident standard errors when there is dependence in the unmodeled dimension. For example, when there is temporal error autocorrelation ($\delta \neq 0$), we see the standard errors of the SEM model is overconfident (i.e., Avg. SE < SD) and, as a result, the coverage probabilities are lower than the targeted 95%. Third, the LDV and SAR models perform poorly under error dependence, as this is now partially captured by the (time or spatial) lag of the outcome, producing an inflationary bias in ϕ and ρ and consequently

[6] All other model features are held fixed and identical to the DGP in the main text: $\beta = 2$, $\mathbf{x}$ is generated with spatial and temporal dependence, etc.

an attenuating bias in β (due to the covariate between the lags and $\mathbf{x}$).[7] This can be seen most acutely in the performance of the LDV model under higher values of δ, as the bias increases and the coverage probabilities decrease. Fortunately, the STADL model performs well across all simulated conditions: unbiased, accurate SEs, low mean square error, and coverage probabilities consistently around 95%.

Table 7: Simulation Results, $\delta = 0.0$

		β					
		Static	SCE	SEM	LDV	SAR	STADL
$\lambda = 0.0$	Bias	-0.001	-0.002	-0.000	-0.001	0.000	-0.002
	Avg. SE	0.011	0.029	0.011	0.027	0.012	0.029
	SD	0.010	0.027	0.010	0.025	0.011	0.027
	MSE	0.000	0.001	0.000	0.001	0.000	0.001
	CP	0.956	0.980	0.956	0.976	0.956	0.976
$\lambda = 0.1$	Bias	-0.000	-0.000	-0.001	0.000	-0.002	0.000
	Avg. SE	0.011	0.029	0.011	0.027	0.012	0.029
	SD	0.010	0.030	0.010	0.028	0.012	0.030
	MSE	0.000	0.001	0.000	0.001	0.000	0.001
	CP	0.964	0.936	0.956	0.936	0.952	0.928
$\lambda = 0.2$	Bias	0.001	0.000	0.001	0.001	-0.003	-0.000
	Avg. SE	0.011	0.029	0.011	0.027	0.012	0.029
	SD	0.012	0.030	0.012	0.028	0.013	0.029
	MSE	0.000	0.001	0.000	0.001	0.000	0.001
	CP	0.936	0.964	0.940	0.940	0.948	0.968
$\lambda = 0.3$	Bias	-0.002	-0.002	-0.002	-0.002	-0.009	-0.001
	Avg. SE	0.011	0.029	0.012	0.027	0.013	0.030
	SD	0.012	0.031	0.011	0.029	0.012	0.029
	MSE	0.000	0.001	0.000	0.001	0.000	0.001
	CP	0.924	0.944	0.972	0.936	0.896	0.948

[7] β in the LDV and SAR models now reflects the short-run and pre-spatial effect respectively, however, under the simulated DGP the true long-run and post-spatial effects should be zero. As such, any bias given here does reflect an underestimation of the short-run, pre-spatial effects and a misattribution of the total effect of $\boldsymbol{x}_t$ on $\boldsymbol{y}_t$.

Table 8: Simulation Results, $\delta = 0.1$

		β					
		Static	SCE	SEM	LDV	SAR	STADL
	Bias	-0.001	0.000	-0.001	-0.030	-0.001	0.000
	Avg. SE	0.011	0.029	0.011	0.027	0.012	0.028
$\lambda = 0.0$	SD	0.011	0.028	0.012	0.025	0.013	0.028
	MSE	0.000	0.001	0.000	0.002	0.000	0.001
	CP	0.940	0.944	0.940	0.808	0.936	0.940
	Bias	0.000	0.000	0.000	-0.029	-0.002	0.000
	Avg. SE	0.011	0.029	0.011	0.027	0.012	0.029
$\lambda = 0.1$	SD	0.013	0.030	0.013	0.026	0.013	0.030
	MSE	0.000	0.001	0.000	0.002	0.000	0.001
	CP	0.900	0.956	0.912	0.776	0.916	0.956
	Bias	-0.001	0.000	-0.001	-0.031	-0.005	-0.001
	Avg. SE	0.011	0.029	0.011	0.027	0.012	0.029
$\lambda = 0.2$	SD	0.012	0.030	0.012	0.028	0.013	0.029
	MSE	0.000	0.001	0.000	0.002	0.000	0.001
	CP	0.932	0.936	0.960	0.792	0.944	0.944
	Bias	0.001	0.001	0.000	-0.032	-0.007	-0.000
	Avg. SE	0.011	0.029	0.012	0.027	0.013	0.030
$\lambda = 0.3$	SD	0.014	0.031	0.014	0.029	0.014	0.029
	MSE	0.000	0.001	0.000	0.002	0.000	0.001
	CP	0.884	0.940	0.908	0.776	0.888	0.956

Table 9: Simulation Results, $\delta = 0.2$

		β					
		Static	SCE	SEM	LDV	SAR	STADL
$\lambda = 0.0$	Bias	-0.001	0.000	-0.001	-0.064	-0.001	0.000
	Avg. SE	0.011	0.029	0.011	0.027	0.012	0.029
	SD	0.013	0.028	0.013	0.026	0.015	0.028
	MSE	0.000	0.001	0.000	0.005	0.000	0.001
	CP	0.900	0.952	0.880	0.352	0.904	0.948
$\lambda = 0.1$	Bias	0.000	0.003	0.000	-0.062	-0.002	0.003
	Avg. SE	0.011	0.029	0.011	0.027	0.012	0.029
	SD	0.014	0.029	0.014	0.026	0.015	0.028
	MSE	0.000	0.001	0.000	0.005	0.000	0.001
	CP	0.876	0.960	0.880	0.336	0.908	0.952
$\lambda = 0.2$	Bias	0.000	-0.001	0.001	-0.066	-0.004	-0.002
	Avg. SE	0.011	0.029	0.012	0.027	0.013	0.029
	SD	0.015	0.029	0.015	0.029	0.015	0.028
	MSE	0.000	0.001	0.000	0.005	0.000	0.001
	CP	0.860	0.948	0.888	0.308	0.880	0.952
$\lambda = 0.3$	Bias	-0.002	-0.002	-0.002	-0.071	-0.010	-0.002
	Avg. SE	0.011	0.029	0.012	0.028	0.013	0.030
	SD	0.016	0.034	0.015	0.031	0.015	0.032
	MSE	0.000	0.001	0.000	0.006	0.000	0.001
	CP	0.824	0.896	0.876	0.308	0.820	0.924

Table 10: Simulation Results, $\delta = 0.3$

		β					
		Static	SCE	SEM	LDV	SAR	STADL
$\lambda = 0.0$	Bias	0.001	0.000	0.001	-0.096	0.001	0.000
	Avg. SE	0.011	0.029	0.011	0.027	0.013	0.028
	SD	0.014	0.029	0.015	0.028	0.017	0.029
	MSE	0.000	0.001	0.000	0.010	0.000	0.001
	CP	0.892	0.932	0.864	0.060	0.860	0.932
$\lambda = 0.1$	Bias	-0.000	-0.002	-0.000	-0.098	-0.002	-0.002
	Avg. SE	0.011	0.029	0.012	0.027	0.013	0.029
	SD	0.016	0.032	0.016	0.031	0.017	0.032
	MSE	0.000	0.001	0.000	0.011	0.000	0.001
	CP	0.836	0.920	0.836	0.080	0.844	0.912
$\lambda = 0.2$	Bias	-0.002	-0.001	-0.002	-0.101	-0.006	-0.001
	Avg. SE	0.011	0.029	0.012	0.028	0.013	0.029
	SD	0.017	0.029	0.016	0.029	0.016	0.028
	MSE	0.000	0.001	0.000	0.011	0.000	0.001
	CP	0.812	0.932	0.856	0.064	0.840	0.944
$\lambda = 0.3$	Bias	-0.001	-0.001	-0.000	-0.110	-0.009	-0.001
	Avg. SE	0.012	0.030	0.012	0.028	0.013	0.030
	SD	0.018	0.034	0.017	0.033	0.017	0.031
	MSE	0.000	0.001	0.000	0.013	0.000	0.001
	CP	0.804	0.912	0.860	0.056	0.808	0.944

Table 11: Simulation Results, $\delta = 0.4$

		β					
		Static	SCE	SEM	LDV	SAR	STADL
	Bias	0.001	-0.001	0.001	-0.137	0.001	-0.001
	Avg. SE	0.012	0.029	0.012	0.028	0.013	0.028
$\lambda = 0.0$	SD	0.016	0.031	0.017	0.030	0.019	0.031
	MSE	0.000	0.001	0.000	0.020	0.000	0.001
	CP	0.844	0.928	0.840	0.012	0.812	0.924
	Bias	0.001	0.000	0.001	-0.137	-0.001	0.000
	Avg. SE	0.012	0.029	0.012	0.028	0.013	0.029
$\lambda = 0.1$	SD	0.020	0.031	0.020	0.032	0.020	0.031
	MSE	0.000	0.001	0.000	0.020	0.000	0.001
	CP	0.736	0.936	0.748	0.004	0.780	0.936
	Bias	-0.002	0.002	-0.002	-0.144	-0.008	0.002
	Avg. SE	0.012	0.029	0.013	0.028	0.013	0.029
$\lambda = 0.2$	SD	0.020	0.030	0.019	0.032	0.020	0.030
	MSE	0.000	0.001	0.000	0.022	0.000	0.001
	CP	0.752	0.960	0.800	0.000	0.780	0.952
	Bias	0.002	0.003	0.001	-0.154	-0.010	0.002
	Avg. SE	0.012	0.029	0.013	0.029	0.014	0.029
$\lambda = 0.3$	SD	0.022	0.031	0.020	0.033	0.020	0.030
	MSE	0.000	0.001	0.000	0.025	0.000	0.001
	CP	0.704	0.924	0.780	0.000	0.760	0.928

Table 12: Simulation Results, $\delta = 0.5$

		β					
		Static	SCE	SEM	LDV	SAR	STADL
$\lambda = 0.0$	Bias	-0.001	0.002	-0.001	-0.186	-0.001	0.002
	Avg. SE	0.012	0.029	0.012	0.029	0.014	0.028
	SD	0.020	0.029	0.020	0.033	0.023	0.029
	MSE	0.000	0.001	0.000	0.036	0.001	0.001
	CP	0.784	0.928	0.784	0.000	0.776	0.928
$\lambda = 0.1$	Bias	-0.000	-0.003	-0.000	-0.192	-0.004	-0.004
	Avg. SE	0.012	0.029	0.013	0.029	0.014	0.029
	SD	0.020	0.031	0.020	0.033	0.022	0.031
	MSE	0.000	0.001	0.000	0.038	0.000	0.001
	CP	0.756	0.928	0.788	0.000	0.800	0.928
$\lambda = 0.2$	Bias	-0.002	0.002	-0.002	-0.197	-0.010	0.002
	Avg. SE	0.012	0.029	0.013	0.030	0.014	0.029
	SD	0.022	0.030	0.021	0.031	0.022	0.029
	MSE	0.000	0.001	0.000	0.040	0.001	0.001
	CP	0.760	0.940	0.800	0.000	0.768	0.960
$\lambda = 0.3$	Bias	0.001	0.000	0.001	-0.223	-0.014	-0.000
	Avg. SE	0.013	0.030	0.014	0.030	0.015	0.030
	SD	0.027	0.034	0.024	0.039	0.024	0.033
	MSE	0.001	0.001	0.001	0.051	0.001	0.001
	CP	0.644	0.920	0.748	0.000	0.708	0.928

3 tscsdep: A New R Package

Among the most significant obstacles to modeling geographical spatial dependence in the analysis of time-series-cross-sectional data, particularly with unbalanced panels, is creating the spatial weights matrix, $\mathbf{W}$. For balanced panels with fixed N and T, this task is relatively easy. One simply pre-Kroenecker-multiplies the spatial weights matrix for a single cross-section by a $T \times T$ identity matrix:

$$\mathbf{W_{NT}} = \mathbf{I_T} \otimes \mathbf{W_N}.$$

Unfortunately, in applied work, it is much more common to have unbalanced panels where T varies across the sample's units. This can arise either due to the entry and exit of units from one's sample (e.g., the recognition of South Sudan), or data missingness. Consequently, the cross-sectional weights matrices are time-period specific—in other words, there are multiple cross-sectional weights matrices associated with a sample. Moreover, because patterns of missingness vary across variables, sample dimensions vary across different regression models. As such, each regression will have a unique TSCS weights matrix. This makes accounting for spatial dependence prohibitively costly.

The package `tscsdep` was created to make it easy to account for (geographical) spatial dependence when working with TSCS data. It draws heavily from the `Cshapes` and `spatialreg` packages. At the moment, there are two main functions in `tscsdep`. The first, `make_ntspmat`, generates a nearest neighbor spatial weights matrix for an unbalanced TSCS sample of countries observed annually, which was used to estimate a non-spatial linear regression model. The call to execute the function is

```
wm <- make_ntspmat(lmobj,ci,yi,k)
```

In this call, `wm` stores the output, the weights matrix; `lmobj` is an object created by the `lm` function. This object contains information about the data and regression specification; `ci` and `yi` are names of variables that identify the country name and year for each observation in the sample;

and `k` is the number of nearest neighbors used for the spatial weights. The second function, `ntspreg`, re-estimates the original linear regression with a spatial lag using the TSCS weights matrix created by `make_ntspmat`. The call is

```
sar <- ntspreg(lmobj,wm)
```

In this line, `sar` stores the spatial regression output; `lmobj` is the same `lm` object used to create the TSCS spatial weights matrix, `wm`, which is also included as the second and final argument in the function. The package is available at https://github.com/MTSS-Textbook/tscsdep.

3.1 Political Accountability / Infant Mortality Reanalysis

In their forthcoming *APSR* article, Lührmann, Marquardt and Mechkova (n.d.) develop several new country-year indices of vertical, horizontal and diagonal political accountability, as well as an overall index of accountability. Much of their article is devoted to demonstrating the content, convergent and construct validity of these measures. For construct validity, the authors show that their overall index of political accountability correlates with infant mortality rates: higher accountability is associated with lower infant mortality. They estimate four time-series-cross-sectional regressions using their overall index of political accountability, both in isolation and in combination with alternative measures of accountability taken from the World Bank and Freedom House.

In this section, we conduct a brief reanalysis of their first, and probably most important, regression using our new R package, `tscsdep`. To illustrate an important feature of the `make_ntspmat` function, we start by assuming the country names in the Lührmann et al. (n.d.) dataset match COW country names, which are used as identifiers for the construction of the spatial-weights matrix.

```
data<-read.csv("accountability_data_regressions.csv")


reg<-lm(formula = infant ~ Accountability + aid + loggdp + gdp_grow +
```

```
    resourcesdep_hm + gini2 + lnpop + urban_cow + violence_domestic +
    communist + rx_infant + v2x_corr + as.factor(country_name) + as.factor(year), data = data)
wm <- make_ntspmat_ch(reg,country_name,year,5)
```

We read the data, estimate the non-spatial linear regression, and then run the `make_ntspmat` function, hoping to create a nearest neighbor spatial weights matrix, using the five nearest neighbors. Unfortunately, the initial output tells us that the country names are not perfectly matched in two different ways. In the first two cases, the problem is that the start dates differ between the dataset and COW. Macedonia enters the dataset in 1993, but CShapes does not have a separate shapefile for Macedonia until 1994. In the remaining seven cases (except Kosovo), the problem is with the name, and these need to be corrected before estimating the spatial-lag regression. For example Burma/Myanmar should be just Myanmar, and Korea, South should be South Korea. The COW country names are available at https://correlatesofwar.org/data-sets/cow-country-codes.

	Data Country Name	Data Start Year	COW Country Name	COW Start Year
1	Macedonia	1993	Macedonia	1994
2	Serbia	1983	Serbia	2006
3	Burma/Myanmar	1967		
4	Congo, Democratic Republic of	2006		
5	Congo, Republic of the	1968		
6	Gambia	1975		
7	Ivory Coast	1966		
8	Korea, South	1960		
9	Kosovo	2000		

In the next chunk of code, we correct the country names to match COW, using the `recode_factor` function. In the case of Serbia, we change the name to Yugoslavia for the entire sample even though CShapes has a separate shapefile for Serbia starting in 2006, the last year of the sample. In the case of Macedonia we just need to change the country name to Yugoslavia for 1993. CShapes does not have a separate shapefile for Kosovo until after the sample period ends, so its country name is changed to Yugoslavia for the entire sample period.

```
data<-read.csv("accountability_data_regressions.csv")


data$country_name<-recode_factor(data$country_name,"Burma/Myanmar"="Myanmar")
data$country_name<-recode_factor(data$country_name,"Korea, South"="South Korea")
data$country_name<-recode_factor(data$country_name,"Kosovo"="Yugoslavia")
data$country_name<-recode_factor(data$country_name,"Serbia"="Yugoslavia")
data[data$country_name=="Macedonia" & data$year=="1993",]$country_name<-"Yugoslavia"
data$country_name<-recode_factor(data$country_name,"Ivory Coast"="Cote d'Ivoire")
data$country_name<-recode_factor(data$country_name,"Congo, Democratic Republic of"="Congo, DRC")
data$country_name<-recode_factor(data$country_name,"Congo, Republic of the"="Congo")
data$country_name<-recode_factor(data$country_name,"Gambia"="The Gambia")


reg <-lm(formula = infant ~ Accountability + aid + loggdp + gdp_grow +
    resourcesdep_hm + gini2 + lnpop + urban_cow + violence_domestic +
    communist + rx_infant + v2x_corr + as.factor(country_name) + as.factor(year), data = data)


wm <- make_ntspmat_ch(reg,country_name,year,5)
```

After the country names are matched, we re-estimate the regression. Note that this regression replicates the original analysis in Lührmann et al. (n.d.). Their first infant mortality regression (MODEL 1 in Figure 8) includes the new overall accountability index and a full set of controls.

```
Call:
lm(formula = infant ~ Accountability + aid + loggdp + gdp_grow +
    resourcesdep_hm + gini2 + lnpop + urban_cow + violence_domestic +
    communist + rx_infant + v2x_corr + as.factor(country_name) +
    as.factor(year), data = data)


Residuals:
    Min      1Q  Median      3Q     Max
```

```
-35.697  -4.447  -0.134   4.175  54.008

Coefficients:
                                                 Estimate Std. Error t value Pr(>|t|)
(Intercept)                                     177.17484   13.36142  13.260  < 2e-16 ***
Accountability                                   -4.25612    0.35093 -12.128  < 2e-16 ***
aid                                              -0.04820    0.03094  -1.558 0.119342
loggdp                                           -9.55947    0.76112 -12.560  < 2e-16 ***
gdp_grow                                          0.03271    0.02357   1.387 0.165372
resourcesdep_hm                                   0.03927    0.02165   1.814 0.069784 .
gini2                                            -0.06158    0.03090  -1.993 0.046376 *
lnpop                                           -13.87935    1.48460  -9.349  < 2e-16 ***
urban_cow                                        -0.14241    0.02798  -5.090 3.75e-07 ***
violence_domestic                                 0.35836    0.12857   2.787 0.005339 **
communist                                         0.95563    1.62036   0.590 0.555382
rx_infant                                         0.67381    0.01952  34.515  < 2e-16 ***
v2x_corr                                         -2.90670    1.90546  -1.525 0.127221

---
Signif. codes:  0 *** 0.001 ** 0.01 * 0.05 . 0.1   1

Residual standard error: 8.695 on 4150 degrees of freedom
  (12955 observations deleted due to missingness)
Multiple R-squared:  0.9609,Adjusted R-squared:  0.959
F-statistic: 502.4 on 203 and 4150 DF,  p-value: < 2.2e-16
```

Once we have replicated the original analysis, we call `make_ntspmat` to create the nearest-neighbor weights matrix (using distance between capital cities to calculate distance). If the country names are, in fact, matched correctly, the console output will change.

```
[1] 1960
 [1]  70 380 740 290 140   2 750 732  20 220  90 645 325 600 210  95 200 780 390 375 350 820 920 385
```

```
[25] 310
[1] All of your Countries are Matched.
[1] 1961
 [1] 70  380 740 290 140 2   92  135 750 732 20  220 90  645 325 600 210 95  616 200 165 780 390 375
[25] 350 820 920 385 310
[1] All of your Countries are Matched.
.
.
.
[1] 2006
 [1] 70  452 560 365 339 651 100 290 140 92  771 145 91  432 770 135 625 700 160 475 840 510 800 500
[25] 101 760 439 811 541 790 436 551 438 437 435 516 482 155 94  130 90  645 663 367 450 712 600 95
[49] 640 369 165 371 373 370 471 490 42  372 705 703 812 580 359 517 780 572 702 461 346 355 344 950
[73] 368 343 820 150 360 317 310
[1] All of your Countries are Matched.
```

The year and COW country codes for the corresponding cross-section will be printed to the console. When finished, the function `make_ntspmat` returns a list. The first element of the list is the original dataset. The second element is the desired $NT \times NT$ weights matrix. The last element is a vector of unique country-year identifiers.

In the chunk of code below, we extract the second element of this list, the nearest-neighbor weights matrix, and run the function `ntspreg`, which returns a list of output from the function `lagsarlm`. The nearest-neighbors weights matrix is automatically row-standardized by `ntspreg`.

```
w <- as.matrix(wm[[2]])
sar <- ntspreg(reg,w)
summary(sar)
```

The results are summarized below.

```
Call:spatialreg::lagsarlm(formula = formula, data = df, listw = listw,
    method = "eigen", zero.policy = TRUE, tol.solve = 1e-11)
```

```
Residuals:
      Min        1Q    Median        3Q       Max
-35.84769  -4.45792  -0.13528   4.17203  53.71579

Type: lag
Coefficients: (asymptotic standard errors)
                                                  Estimate Std. Error  z value  Pr(>|z|)
(Intercept)                                     176.385836  13.042802  13.5236 < 2.2e-16
Accountability                                   -4.253048   0.342422 -12.4205 < 2.2e-16
aid                                              -0.048687   0.030190  -1.6127 0.1068100
loggdp                                           -9.534110   0.742749 -12.8362 < 2.2e-16
gdp_grow                                          0.033456   0.023006   1.4542 0.1458811
resourcesdep_hm                                   0.037964   0.021135   1.7963 0.0724506
gini2                                            -0.062058   0.030154  -2.0580 0.0395855
lnpop                                           -13.903421   1.448662  -9.5974 < 2.2e-16
urban_cow                                        -0.141178   0.027306  -5.1702 2.339e-07
violence_domestic                                 0.366457   0.125541   2.9190 0.0035114
communist                                         0.921072   1.581146   0.5825 0.5602069
rx_infant                                         0.672092   0.019065  35.2519 < 2.2e-16
v2x_corr                                         -2.964505   1.859456  -1.5943 0.1108720

Rho: 0.015671, LR test value: 4.5588, p-value: 0.032749
Asymptotic standard error: 0.0072993
    z-value: 2.1469, p-value: 0.031805
Wald statistic: 4.609, p-value: 0.031805

Log likelihood: -15488.1 for lag model
ML residual variance (sigma squared): 71.989, (sigma: 8.4846)
Number of observations: 4354
Number of parameters estimated: 206
AIC: 31388, (AIC for lm: 31391)
LM test for residual autocorrelation
test value: 94.809, p-value: < 2.22e-16
```

This model only accounts for spatial dependence in the data; temporal dynamics are still ignored. In this regression, the spatial lag coefficient is positive and statistically significant, suggesting (nearest-neighbor) spatial dependence in the data beyond that captured by the authors' regional average infant mortality variable (rx_infant).

We include our replication of the original results in the first column of Table 1. The estimates from our reanalysis regression incorporating spatially and temporally lagged dependent–variable regressors are presented in the second column. The long-run effects of all the variables measured in levels, the variables included in $\mathbf{x_{it}}$, are provided in the third column. Comparing the results, it is clear that our concerns were warranted. Comparing the (total) spatial effect in the original regression which ignores temporal (autoregressive) dynamics and regional common shocks with our long-run spatial effect, we estimate that the former overstates the true degree of long-run spatial dependence by nearly 38%. This has consequences. Again, comparing the first and third columns, we estimate that the effect of political accountability on infant mortality rates is more double the size of the effect estimated by Lührmann et al. (n.d.).

An alternative strategy to account for regional common shocks is to add spatial lags in first differences to regression models. Because spatial lags represent autoregression in space—countries have first, second and third (etc.) order neighbors—there is some added flexibility with respect to identifying the exact geographical boundaries of shocks, flexibility that does not exist with regional shock indicator variables. (Spatial lags are "leaky;" regional indicators are not.) Additionally, spatial lags can be more parsimonious than regional-period shock indicators because each lag defines a "neighborhood" for every unit in a sample.

More generally, in spatio-temporal autoregressive distributive lag (STADL) models, right-hand-side variables that are differenced produce short-run shocks to left-hand-side outcomes, whereas right-hand-side variables in levels produce long-run effects through temporal multipliers. The Luuml;hrmann et al. regression includes a de facto spatial lag, country and year fixed effects, but no temporal dynamics. We also think that regional shocks are plausible. Therefore, in our

Table 13: Reanalysis of the Accountability / Infant Mortality Regression in Lührmann et al. (n.d.)

	Dependent variable: Infant Mortality		
	Level	Difference	Long Run
Accountability	-4.256^{***}	-0.197^{***}	-9.748
	(0.351)	(0.038)	
Foreign aid	-0.048	0.016^{***}	0.771
	(0.031)	(0.003)	
GDP/capita (ln)	-9.559^{***}	0.763^{***}	37.74
	(0.761)	(0.084)	
Economic Growth	0.033	-0.019^{***}	-0.972
	(0.024)	(0.003)	
Resource dependence	0.040^{*}	0.013^{***}	0.661
	(0.022)	(0.002)	
Economic inequality	-0.062^{**}	0.006	0.293
	(0.031)	(0.003)	
Population (ln)	-13.879^{***}	0.632^{***}	31.23
	(1.485)	(0.163)	
Urbanization	-0.142^{***}	0.023^{***}	1.135
	(0.028)	(0.003)	
Political violence	0.358^{***}	-0.016	-0.810
	(0.129)	(0.014)	
Communist	0.956	-0.784^{***}	-38.77
	(1.620)	(0.174)	
Infant mortality, regional average	0.674^{***}	0.008^{***}	0.419
	(0.020)	(0.002)	
Political corruption index	-2.907^{*}	-0.293	-14.505
	(1.905)	(0.205)	
Temporal Lag (Level)		-0.020^{***}	
		(0.002)	
Spatial Lag (Difference)		0.037^{*}	
		(0.020)	
Observations	4,354	4,312	
Fixed Country Effects	Yes	Yes	
Fixed Year Effects	Yes	Yes	

Note: $^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

reanalysis, we include a temporal lag and a nearest neighbor spatial lag in first differences. The reanalysis regression takes the following form:

$$y_{it} = \mathbf{x_{it}}\boldsymbol{\beta} + \phi y_{it-1} + \rho \mathbf{w_i} \Delta \mathbf{y_t} + f_i + g_t + \varepsilon_{it},$$

where y_{it} is the outcome for unit i at time t, $\mathbf{x_{it}}$ is a $1 \times k$ vector of time t exogenous covariates for unit i, $\boldsymbol{\beta}$ is a $k \times 1$ vector of coefficients, ρ is the spatial lag coefficient, $\mathbf{w_i}$ is a unit specific vector of spatial weights with a zero in the same-unit coordinate, $\Delta \mathbf{y_t}$ is a time t vector of differenced outcomes, f_i is a fixed unit effect, g_t is a fixed period effect and ε_{it} is an *i.i.d.* disturbance for unit i at time t. With some simple algebraic manipulation, the regression can be rewritten with a differenced outcome. For reasons of convenience related to evaluating the likelihood function, this is the regression that we estimate:

$$\Delta y_{it} = \mathbf{x_{it}}\boldsymbol{\beta} + (\phi - 1) y_{it-1} + \rho \mathbf{w_i} \Delta \mathbf{y_t} + f_i + g_t + \varepsilon_{it}.$$

We estimate the Table 1, column 2 STADL in first differences with the code below.

```
library (DataCombine)


data<-read.csv("accountability_data_regressions.csv")


# Lag the variable one time unit by ID group
data <- slide(data = data, Var = 'infant', GroupVar = 'country_id',
NewVar = 'lag_inf', slideBy = -1)


data$diff_inf <- data$infant - data$lag_inf


reg<-lm(formula = diff_inf ~ lag_inf + Accountability + aid + loggdp + gdp_grow +
    resourcesdep_hm + gini2 + lnpop + urban_cow + violence_domestic +
    communist + rx_infant + v2x_corr + as.factor(country_name) + as.factor(year), data = data)


wm2 <- make_ntspmat_ch(reg,country_name,year,5)
w2 <- as.matrix(wm2[[2]])
sar2 <- ntspreg(reg,w2)
```

```
summary(sar2)
```

The summarized results are

```
Call:spatialreg::lagsarlm(formula = formula, data = df, listw = listw,
    method = "eigen", zero.policy = TRUE, tol.solve = 1e-11)

Residuals:
      Min        1Q    Median        3Q       Max
-8.713635 -0.347462  0.024784  0.351405 27.958425

Type: lag
Coefficients: (asymptotic standard errors)
                                                    Estimate  Std. Error  z value  Pr(>|z|)
(Intercept)                                      -10.3920063   1.4866294  -6.9903 2.743e-12
lag_inf                                           -0.0202188   0.0016639 -12.1511 < 2.2e-16
Accountability                                    -0.1970935   0.0382911  -5.1472 2.643e-07
aid                                                0.0155976   0.0033198   4.6983 2.623e-06
loggdp                                             0.7625747   0.0840950   9.0680 < 2.2e-16
gdp_grow                                          -0.0191652   0.0025461  -7.5273 5.174e-14
resourcesdep_hm                                    0.0133679   0.0023253   5.7490 8.979e-09
gini2                                              0.0059318   0.0033512   1.7700 0.0767242
lnpop                                              0.6315082   0.1626417   3.8828 0.0001033
urban_cow                                          0.0229474   0.0030498   7.5243 5.307e-14
violence_domestic                                 -0.0163823   0.0139200  -1.1769 0.2392416
communist                                         -0.7839187   0.1742961  -4.4976 6.872e-06
rx_infant                                          0.0084794   0.0024326   3.4857 0.0004908
v2x_corr                                          -0.2932679   0.2050119  -1.4305 0.1525758

Rho: 0.036807, LR test value: 3.1803, p-value: 0.074532
Asymptotic standard error: 0.019764
    z-value: 1.8623, p-value: 0.062559
Wald statistic: 3.4682, p-value: 0.062559
```

```
Log likelihood: -5810.532 for lag model
ML residual variance (sigma squared): 0.86673, (sigma: 0.93099)
Number of observations: 4312
Number of parameters estimated: 206
AIC: 12033, (AIC for lm: 12034)
LM test for residual autocorrelation
test value: 5.7551, p-value: 0.016441
```

References

Beck, N. and Katz, J. N. (2011), 'Modeling dynamics in time-series-cross-section political economy data', *Annual Review of Political Science* **14**, 331–352.

Lührmann, A., Marquardt, K. L. and Mechkova, V. (n.d.), 'Constraining governments: New indices of vertical, horizontal, and diagonal accountability', *American Political Science Review* .