April 2016, Problem 2. Let $f\colon \mathbb{R} \to \mathbb{R}$ be a C^4 function such that for all $x, h \in \mathbb{R}$,

$$f(x+h) = f(x) + f'(x)h + \frac{1}{2}f''(x+\frac{1}{3}h)h^2.$$

Show that the fourth derivative $f^{(4)}(x) = 0$ for any $x \in \mathbb{R}$.