April 2013, Problem 2. Let f and g be two Riemann integrable functions on $[0,1]$, and $h(x) = \max\{f(x), g(x)\}$ for $x \in [0,1]$.

(i) Prove that h is Riemann integrable on $[0,1]$;

(ii) Suppose that $\{f_n\}$ and $\{g_n\}$ are two sequences of Riemann integrable functions on $[0,1]$ such that

$$\lim_{n\to\infty} \int_0^1 |f_n(x) - f(x)|\, dx = \lim_{n\to\infty} \int_0^1 |g_n(x) - g(x)|\, dx = 0$$

Let $h_n(x) = \max\{f_n(x), g_n(x)\}$ for $x \in [0,1]$ and $n \in \mathbb{N}$. Prove that

$$\lim_{n\to\infty} \int_0^1 |h_n(x) - h(x)|\, dx = 0$$