April 2011, Problem 2. Let (M, d) be a metric space such that

$$d(x, z) \leq \max\{d(x, y), d(y, z)\}$$

for all $x, y, z \in M$. For any $x \in M$ and $r > 0$, the set $B(x, r) = \{y \in M : d(y, x) < r\}$ is called an open ball in M.

(a) Prove that every open ball in M is a closed set.

(b) Prove that if two open balls in M have a common point, then one of them is contained in the other.

(*Contextual note.* A metric with the property above is called an *ultrametric*. These can be found "in nature": for example the *p-adic norm* on $\mathbb{Q}$, p prime, (google it) is an example.)