Pugh Ed. 2, Ch. 2, #46. Assume that A, B are compact, disjoint, nonempty subsets of M. Prove that there are $a_0 \in A$ and $b_0 \in B$ such that for all $a \in A$ and $b \in B$ we have

$$d(a_0, b_0) \leq d(a, b).$$

[The points a_0, b_0 are closest together.]

August 2012, Problem 1. Let (M, d) be a compact metric space and $z \in M$. Let $T\colon M \to M$ be a function which satisfies $d(x, y) \leq d(T(x), T(y))$ for all $x, y \in M$, i.e. the distances are non-decreasing under the mapping T. Define $\{x_n\}$ by

$$x_1 = T(z) \quad \text{and } x_{n+1} = T(x_n) \text{ for } n \geq 1.$$

Prove that there exists a subsequence of $\{x_n\}$ which converges to z.