**Pugh Ed. 2, Ch. 2, #22.** If every closed and bounded subset of a metric space $M$ is compact, does it follow that $M$ is complete? (Proof or counterexample.)

**August 2015, Problem 3.** Suppose that a set $A \subset \mathbb{R}^n$ is a union of an increasing family of compact sets $A = \bigcup_{i=1}^{\infty} A_i$, $A_1 \subset A_2 \subset \ldots$ Suppose also that there is a compact set $C \subset \mathbb{R}^n$ such that

$$\forall i \in \mathbb{N}\ \forall x \in A \backslash A_i \quad \operatorname{dist}(x, C) < \frac{1}{i}.$$

Prove that the closure of the set $A$ satisfies $\overline{A} \subset A \cup C$.